# Lesson 11: Completing the Square

Rewrite each expression by completing the square.

a. $p^2 - 2p - 48$

$$p^2 - 2p - 48 = p^2 - 2p + \_\_ - 48 - \_\_$$
$$= p^2 - 2p + 1 - 48 - 1$$
$$= (p-1)^2 - 49$$

I could use the tabular method to help me complete the square and factor.

When the coefficient of the squared term is 1, the number that completes the square is $\frac{1}{2}$ of the linear term squared. In this problem, $\left(\frac{1}{2} \cdot -2\right)^2 = 1$.

b. $y^2 - 7y - 52$

$$y^2 - 7y - 52 = y^2 - 7y + \_\_ - 52 - \_\_$$
$$= y^2 - 7y + \left(-\frac{7}{2}\right)^2 - 52 - \left(-\frac{7}{2}\right)^2$$
$$= \left(y - \frac{7}{2}\right)^2 - 64.25$$

I can only add 0 to an expression without changing its value, so I have to subtract the number that I add when completing the square.

I am using the equation $a^2 - 2ab + b^2 = (a-b)^2$ to factor this expression.

I could use a calculator to evaluate $-52 - \left(-\frac{7}{2}\right)^2$ or work it by hand: $-52 - \frac{49}{4} = -52 - 12.25 = -64.25$.

# Lesson 12: Completing the Square

**Completing the Square on $ax^2 + bx + c$**

Rewrite each expression by completing the square.

1. $-3x^2 - 9x + 7$

I need to factor the quadratic and linear terms so that the coefficient of the quadratic term is 1.

$$\begin{aligned} -3x^2 - 9x + 7 &= -3(x^2 + 3x) + 7 \\ &= -3(x^2 + 3x + \underline{\quad}) + 7 - \underline{\quad} \\ &= -3\left(x^2 + 3x + \frac{9}{4}\right) + 7 - \left(-3 \cdot \frac{9}{4}\right) \\ &= -3\left(x + \frac{3}{2}\right)^2 + 7 + \frac{27}{4} \\ &= -3\left(x + \frac{3}{2}\right)^2 + \frac{28}{4} + \frac{27}{4} \\ &= -3\left(x + \frac{3}{2}\right)^2 + \frac{55}{4} \end{aligned}$$

I need to add the square of one-half of the $x$-coefficient. The $x$-coefficient is 3.

$$\left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

I have to subtract whatever number I add to make sure this expression is equal to the original expression.

The first part of the expression factors using the identity $(a + b)^2 = a^2 + 2ab + b^2$.

I need to rewrite the rest of the expression as a single numerical value.

2. $100x^2 - 65x + 80$

To factor out 100, I need to divide 65 by 100.

$$\begin{aligned} 100x^2 - 65x + 80 &= 100(x^2 - 0.65x + \underline{\quad\quad}) + 80 - \underline{\quad} \\ &= 100(x^2 - 0.65x + 0.105625) + 80 - 100(0.105625) \\ &= 100(x - 0.325)^2 + 69.4375 \end{aligned}$$

I can use a calculator to help me compute $\left(\frac{-0.65}{2}\right)^2 = 0.105625$.

I have to remember to multiply by the number I factored out of the first two terms.



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# Lesson 13: Solving Quadratic Equations by Completing the Square

Solve the equation by completing the square.

$3x^2 = 9x + 7$

I need to rewrite the equation so that the $x$ terms are on one side and the constant term is on the other side of the equal sign.

$$3x^2 = 9x + 7$$
$$3x^2 - 9x = 7$$
$$3\left(x^2 - 3x + \left(-\frac{3}{2}\right)^2\right) = 7 + 3\left(-\frac{3}{2}\right)^2$$

I use the addition property of equality to add the number that completes the square to both sides of the equation.

$$3\left(x - \frac{3}{2}\right)^2 = \frac{55}{4}$$

Before taking the square root of both sides, I need to divide both sides by 3, which is the same as multiplying both sides by $\frac{1}{3}$.

$$\left(x - \frac{3}{2}\right)^2 = \frac{55}{4} \cdot \frac{1}{3}$$

I recall that there are two solutions to equations of the form $x^2 = b$.

The solutions are $\sqrt{b}$ and $-\sqrt{b}$.

$$x - \frac{3}{2} = +\sqrt{\frac{55}{12}} \text{ or } x - \frac{3}{2} = -\sqrt{\frac{55}{12}}$$
$$x = \frac{3}{2} + \sqrt{\frac{55}{12}} \text{ or } x = \frac{3}{2} - \sqrt{\frac{55}{12}}$$

*Both solutions are irrational numbers.*

*The exact solution set is* $\left\{\frac{3}{2} + \sqrt{\frac{55}{12}}, \frac{3}{2} - \sqrt{\frac{55}{12}}\right\}$.

I could rewrite the square root expression so no fractions or perfect square factors appear under the square root.

*Rounded to the nearest hundredth, the solution set is* $\{3.64, -0.64\}$.

# Lesson 14: Deriving the Quadratic Formula

## Using the Quadratic Formula

Solve each equation using the quadratic formula.

1. $-3x^2 - 9x + 7 = 0$

   *$a = -3$, $b = -9$, and $c = 7$*

I need the values of $a$, $b$, and $c$ to substitute into the formula.

I can substitute first and then work on rewriting the expression.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-9) \pm \sqrt{(-9)^2 - 4(-3)(7)}}{2(-3)}$$

$$x = \frac{9 \pm \sqrt{81 + 84}}{-6}$$

$$x = \frac{9 \pm \sqrt{165}}{-6}$$

The $\pm$ reminds me that there are two solutions to this equation.

$$x = -\frac{9}{6} + \frac{\sqrt{165}}{6} \text{ or } x = -\frac{9}{6} - \frac{\sqrt{165}}{6}$$

I can rewrite $\frac{9}{6}$ as $\frac{3}{2}$ because 9 and 6 have a common factor, but I cannot rewrite the radical part of the expression because the irrational number $\sqrt{165}$ and 6 do not have a common factor.

*The solution set is* $\left\{-\frac{3}{2} + \frac{\sqrt{165}}{6}, -\frac{3}{2} - \frac{\sqrt{165}}{6}\right\}$.

The exact solutions are irrational numbers. I could use a calculator to find decimal approximations of these numbers as well.



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2. $x^2 + \frac{1}{2}x + 2 = \frac{31}{16}$

$$x^2 + \frac{1}{2}x + 2 - \frac{31}{16} = 0$$

$$x^2 + \frac{1}{2}x + \frac{1}{16} = 0$$

I need to rearrange the equation so that one side is 0 and the other has the expression in standard form $ax^2 + bx + c$.

*From the equation, $a = 1$, $b = \frac{1}{2}$, and $c = \frac{1}{16}$.*

$$x = \frac{-\left(\frac{1}{2}\right) \pm \sqrt{\left(\frac{1}{2}\right)^2 - 4(1)\left(\frac{1}{16}\right)}}{2(1)}$$

$$x = \frac{-\frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{1}{4}}}{2}$$

$$x = \frac{-\frac{1}{2} \pm \sqrt{0}}{2}$$

$$x = -\frac{1}{2} \div 2$$

The formula does not change when fractions or decimals are involved, but rewriting the expression can be more difficult.

This equation will only have one solution because the square root of 0 is 0.

*The solution is $\left\{-\frac{1}{4}\right\}$.*

If I divide $-\frac{1}{2}$ into two equal parts, each part is $-\frac{1}{4}$.

# Lesson 15: Using the Quadratic Formula

## Using the Discriminant

Without solving, determine the number of real solutions for each quadratic equation.

1. $2x^2 - 8x + 3 = 0$

   $a = 2, b = -8, c = 3$

   *The discriminant is* $(-8)^2 - 4(2)(3) = 64 - 24 = 40$.

   *The discriminant is positive, so there are* $2$ *real solutions.*

I need to analyze the sign of the expression under the radical in the quadratic formula, which is $b^2 - 4ac$.

2. $3 + 3n = -4n^2 + 2$

I can add $4n^2$ and subtract 2 from both sides of the equation.

$$4n^2 + 3n + 3 - 2 = 0$$
$$4n^2 + 3n + 1 = 0$$

It is easiest to identify the values of $a$, $b$, and $c$ when the equation is in the form $ax^2 + bx + c = 0$.

   $a = 4, b = 3, c = 1$

   *The discriminant is* $(3)^2 - 4(4)(1) = 9 - 16 = -7$.

   *The discriminant is negative, so there are no real solutions.*

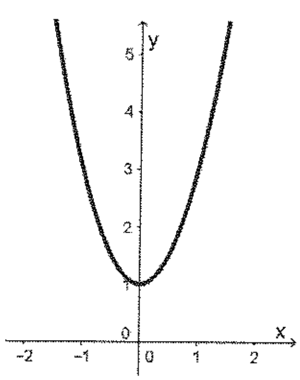
I also recall that if the discriminant is equal to 0, then there is one real solution.

The number of real solutions is equal to the number of $x$-intercepts, which are also called zeros or roots.

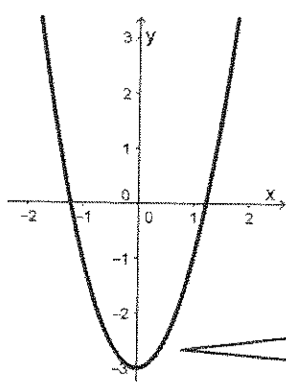
## Connecting Solutions to $f(x) = 0$ with the Graph of $y = f(x)$

3. Based on the graph of $y = f(x)$ shown below, determine the number of real solutions for each corresponding equation, $f(x) = 0$.

a.




b.




*Part (a) has no real solutions.*

*Part (b) has two real solutions.*

I need to see how many times the graph touches the $x$-axis.

I recall doing a problem like this on Exercise 11 on my student pages.



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**Writing a Quadratic in Factored Form**

4. Consider the quadratic function $f(x) = 2x^2 - 3$.

   a. Find the $x$-intercepts of the graph of the function.

$$2x^2 - 3 = 0$$
$$2x^2 = 3$$
$$x^2 = \frac{3}{2}$$
$$x = \sqrt{\frac{3}{2}} \text{ or } x = -\sqrt{\frac{3}{2}}$$

I need to make the function equal to 0.
I could have used the quadratic formula to solve this with $a = 2$, $b = 0$, and $c = -3$.

There are two solutions to equations of the form $x^2 = b$ where $b$ is greater than 0.

*The x-intercepts are* $\sqrt{\frac{3}{2}}$ *and* $-\sqrt{\frac{3}{2}}$.

   b. Use the $x$-intercepts to write the quadratic function in factored form.

$$f(x) = 2\left(x - \sqrt{\frac{3}{2}}\right)\left(x - \left(-\sqrt{\frac{3}{2}}\right)\right) \text{ or } f(x) = 2\left(x - \sqrt{\frac{3}{2}}\right)\left(x + \sqrt{\frac{3}{2}}\right)$$

The form is $f(x) = a(x - m)(x - n)$ where $m$ and $n$ are the $x$-intercepts.

   c. Show that the function from part (b) written in factored form is equivalent to the original function.

$$f(x) = 2\left(x - \sqrt{\frac{3}{2}}\right)\left(x + \sqrt{\frac{3}{2}}\right)$$
$$= 2\left[\left(x - \sqrt{\frac{3}{2}}\right)(x) + \left(x - \sqrt{\frac{3}{2}}\right)\left(\sqrt{\frac{3}{2}}\right)\right]$$
$$= 2\left(x^2 - \sqrt{\frac{3}{2}}x + \sqrt{\frac{3}{2}}x - \left(\sqrt{\frac{3}{2}}\right)^2\right)$$
$$= 2\left(x^2 - \frac{3}{2}\right)$$
$$= 2x^2 - 3$$

I can use the distributive property repeatedly to rewrite this expression.

The middle two terms make a 0.

# Lesson 16: Graphing Quadratic Equations from the Vertex Form, $y = a(x - h)^2 + k$

I can think of $(x + 1.1)$ as $\big(x - (-1.1)\big)$ to help me identify the $x$-coordinate of the vertex.

## The Vertex Form of a Quadratic Equation

1. Find the coordinates of the vertex of the graph of the quadratic equation $y = 2(x + 1.1)^2 + 3$.

   *The vertex coordinates are* $(-1.1, 3)$.

2. Write a quadratic equation to represent a function with a vertex at $(2, -3)$. Use a leading coefficient other than 1.

   *The vertex coordinates are* $(h, k)$. *In this problem,* $h = 2$ *and* $k = -3$. *Let* $a = -2$.

   *Using* $y = a(x - h)^2 + k$, *substitute the values of a, h, and k into the formula. The equation is*

   $$y = -2(x - 2)^2 + (-3) \text{ or } y = -2(x - 2)^2 - 3.$$

3. Use vocabulary from this lesson (i.e., *stretch, shrink, opens up,* and *opens down*) to compare and contrast the graphs of the quadratic equations $y = -x^2 - 2$ and $y = 3x^2 + 2$.

   *The graph of* $y = -x^2 - 2$ *opens downward with the vertex at* $(0, -2)$, *and the graph of* $y = 3x^2 + 2$ *opens upward and has a vertical stretch by a factor of* 3 *compared to the other graph and has a vertex at* $(0, 2)$.

In the equation, I can think of $x^2$ as $(x - 0)^2$ to remind me that the $x$-coordinate of the vertex is 0.

I know if the leading coefficient is negative, then the graph opens downward.

# Lesson 17: Graphing Quadratic Functions from the Standard Form, $f(x) = ax^2 + bx + c$

## Graphing Quadratic Functions in Standard Form

The lesson summary details the steps to graph a quadratic function.

1. Graph $f(x) = -x^2 - 3x + 18$, and describe the key features.

*The graph opens downward because $a$ is negative. The $y$-intercept is $18$ because $f(0) = -0^2 - 3(0) + 18 = 18$. To find the $x$-intercepts, solve $f(x) = 0$ for $x$.*

$$-x^2 - 3x + 18 = 0$$
$$-(x^2 + 3x - 18) = 0$$
$$-(x + 6)(x - 3) = 0$$
$$x + 6 = 0 \textit{ or } x - 3 = 0$$

I can factor the quadratic. I could also solve this equation by completing the square or using the quadratic formula.

*The solutions to the equation are $-6$ and $3$. These are the $x$-intercepts.*

*Find the vertex using symmetry. The distance between $-6$ and $3$ is $9$ units.*

*Add half of this number to $-6$ to find the $x$-coordinate of the vertex.*

$$\frac{9}{2} + (-6) = -1.5$$

I could also recall that when a quadratic is written in standard form, the $x$-coordinate is $-\frac{b}{2a}$.

*Find the value of $f(-1.5)$ to get the $y$-coordinate of the vertex.*

$$f(-1.5) = -(-1.5)^2 - 3(-1.5) + 18$$
$$= -2.25 + 4.5 + 18$$
$$= 20.25$$

*The coordinates of the vertex are $(-1.5, 20.25)$.*

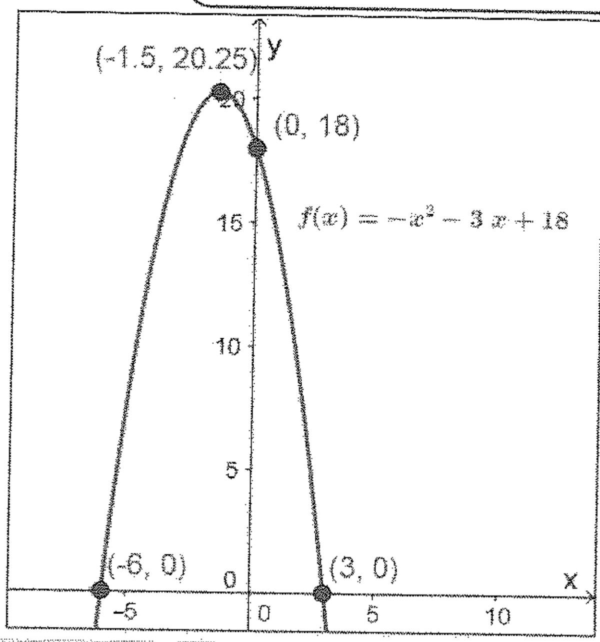
*The axis of symmetry is $x = -1.5$.*

*Plot the intercepts and the vertex.*

*Draw a smooth curve through these points to complete the graph of the function.*

*End behavior: $x \to \pm\infty, f(x) \to -\infty$*

Because the graph opens downward, the end behavior of the function is approaching negative infinity.



## Profit Functions

The function $p(x) = -0.15x^2 + 25x - 250$ gives the profit in dollars that the swim club makes from offering a summer swim camp priced at $x$ dollars per camper, $p(x)$.

2. What is the start-up cost for the camp?

   *Calculate* $p(0) = -0.15(0)^2 + 25(0) - 250 = -250.$

   *The start-up cost is* $\$250$.

   If they set the price at \$0, they will have a negative profit of \$250.

3. What price should they charge to make a profit?

   *The* $x$*-intercepts are the solutions to the equation* $p(x) = 0$. *The graph of* $p$ *opens downward, so* $p(x)$ *will be greater than* $0$ *between the* $x$*-intercepts.*

$$-0.15x^2 + 25x - 250 = 0$$

   *Using the quadratic formula*

$$x = \frac{-25 + \sqrt{(25)^2 - 4(-0.15)(-250)}}{2(-0.15)} \approx 10.69 \text{ and}$$

$$x = \frac{-25 - \sqrt{(25)^2 - 4(-0.15)(-250)}}{2(-0.15)} \approx 155.98$$

   I can look back at Module 4 Lesson 15 if I forget the formula. I can use a calculator to find the approximate value of these expressions.

   *They will need to charge at least* $\$11$ *and not more than* $\$155$ *to make a profit.*

4. What price should they charge to make the most profit?

   I could also graph the profit function on a calculator and determine the maximum value by tracing the graph, or I could complete the square on the expression $-0.15x^2 + 25x - 250$ to rewrite the expression in vertex form.

   *Find the vertex of the graph of* $p$.

   *From the function,* $a = -0.15$ *and* $b = 25$.

   *The* $x$*-coordinate of the vertex is given by the formula* $x = -\frac{b}{2a}$.

$$x = -\frac{25}{2(-0.15)} = -\frac{25}{-0.3} = \frac{250}{3} = 83\frac{1}{3}$$

   *It makes sense to price the camp to the nearest dollar.*

   *Evaluate* $p(83)$ *and* $p(84)$ *to determine which gives the larger dollar amount.*

$$p(83) = -0.15(83)^2 + 25(83) - 250 = 791.65$$

$$p(84) = -0.15(84)^2 + 25(84) - 250 = 791.60$$

   *If they set the price at* $\$83$ *per camper, they will make* $\$791.65$ *in profit.*





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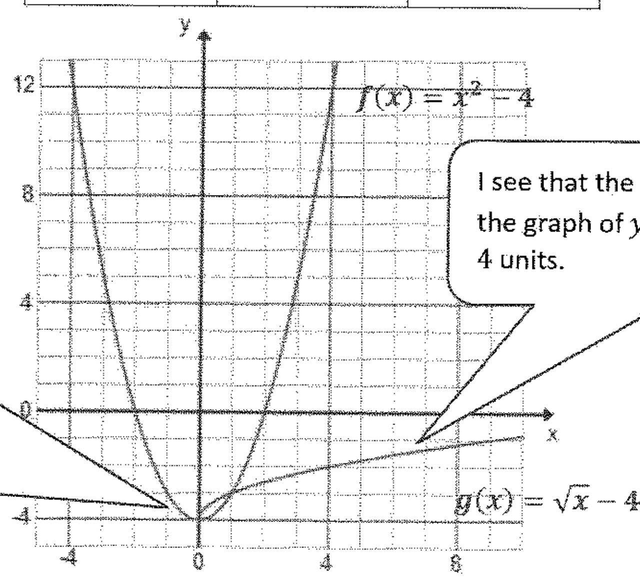
# Lesson 18: Graphing Cubic, Square Root, and Cube Root Functions

## Graphing a Square Root Function

1. Create the graphs of the functions $f(x) = x^2 - 4$ and $g(x) = \sqrt{x} - 4$ using the given values. Use a calculator to help with decimal approximations.

| $x$ | $f(x)$ | $g(x)$ |
|---|---|---|
| $-4$ | $12$ | *Error* |
| $-2$ | $0$ | *Error* |
| $-1$ | $-3$ | *Error* |
| $0$ | $-4$ | $-4$ |
| $1$ | $-3$ | $-3$ |
| $2$ | $0$ | $\approx -2.5858$ |
| $4$ | $12$ | $-2$ |

I know that the domain of $g$ is limited to nonnegative numbers since the square root of a negative number is not real.



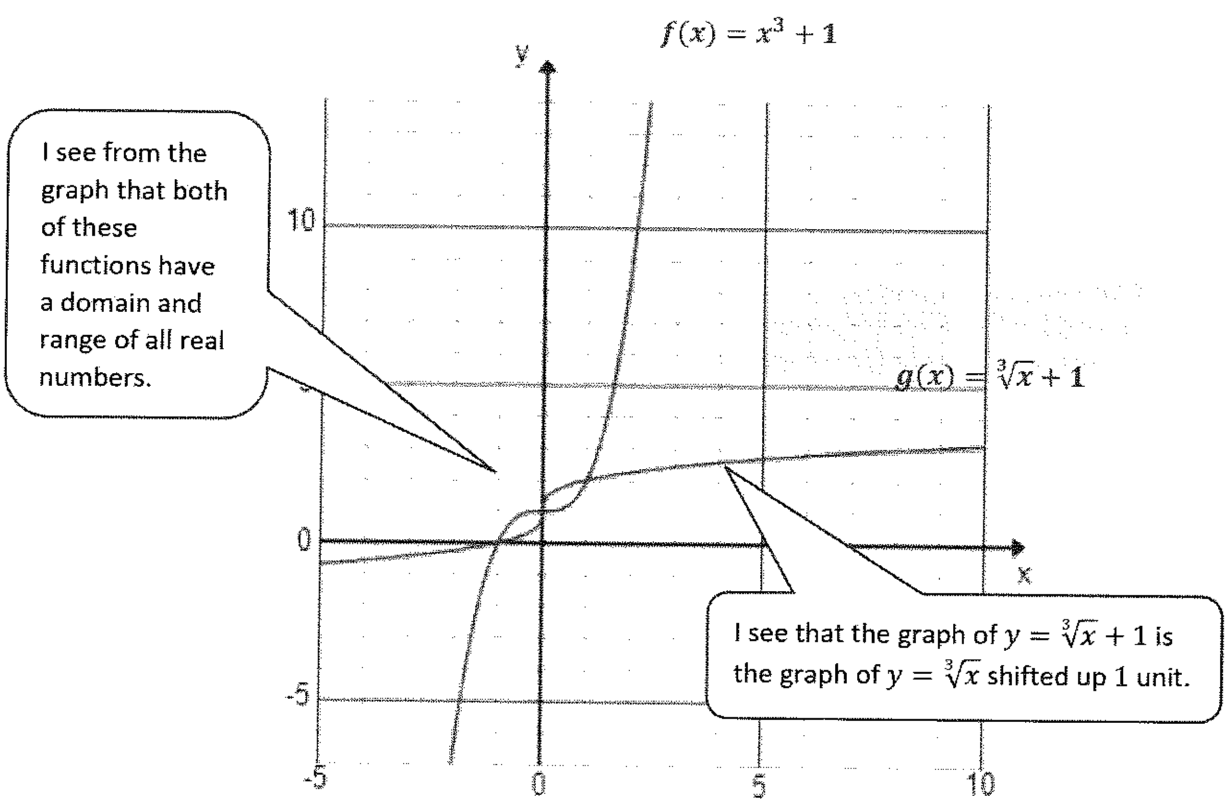

I see from the graph that while these two functions have different domains, they have the same range ($y \geq -4$).

I see that the graph of $y = \sqrt{x} - 4$ is the graph of $y = \sqrt{x}$ shifted down 4 units.

## Graphing a Cubic and a Cube Root Function

2. Create the graphs of the functions $f(x) = x^3 + 1$ and $g(x) = \sqrt[3]{x} + 1$ using the given values. Use a calculator to help with decimal approximations.

| $x$ | $f(x)$ | $g(x)$ |
|---|---|---|
| $-8$ | $-511$ | $-1$ |
| $-2$ | $-7$ | $\approx -0.2599$ |
| $-1$ | $0$ | $0$ |
| $0$ | $1$ | $1$ |
| $1$ | $2$ | $2$ |
| $2$ | $9$ | $\approx 2.2599$ |
| $8$ | $513$ | $3$ |



# Lesson 19: Translating Graphs of Functions

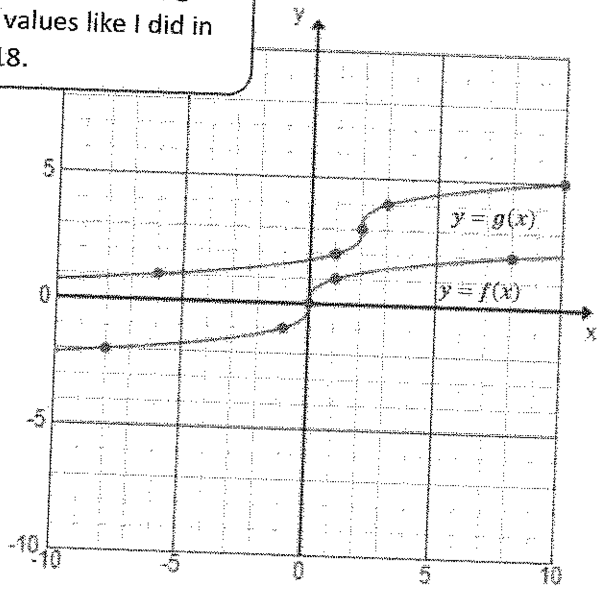
1. Graph the following two functions on the same coordinate plane.

$f(x) = \sqrt[3]{x}$

$g(x) = \sqrt[3]{x-2} + 3$

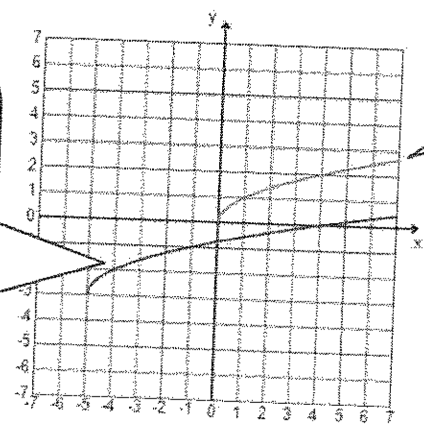
I can graph $f$ by making a table of values like I did in Lesson 18.

I can graph $g$ by using what I know about transformations of functions. The graph of $y = g(x)$ will be the graph of $y = f(x)$ shifted 2 units right and 3 units up.




2. Study the graphs below. Identify the parent function and the transformations of that function depicted by the second graph. Then, write the formula for the transformed function.

I see that this graph is a transformation of the graph of the parent function, $f(x) = \sqrt{x}$.




I recognize this graph from Lesson 18. It is the graph of $f(x) = \sqrt{x}$.

*The parent function is $f(x) = \sqrt{x}$. Its graph is the one with an endpoint at $(0, 0)$. The second graph is the graph of the parent function shifted left 5 units and down 3 units. The formula for the translated function is $g(x) = \sqrt{x+5} - 3$.*

# Lesson 20: Stretching and Shrinking Graphs of Functions

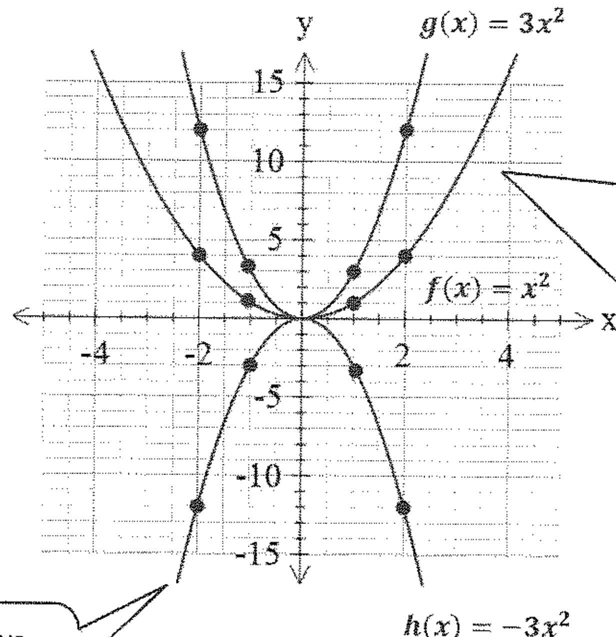
1. Graph the following functions on the same coordinate plane.

   $f(x) = x^2$

   $g(x) = 3x^2$

   $h(x) = -3x^2$

> This is the parent function. I know that the graphs of $g$ and $h$ are transformations of the graph of $f$.




> I can graph $y = g(x)$ by multiplying the $y$-value of each coordinate on the graph of $y = f(x)$ by 3. The graph of $y = g(x)$ is the graph of $y = f(x)$ stretched vertically by a scale factor of 3.

> I know that the negative will cause the graph to reflect across the $x$-axis.



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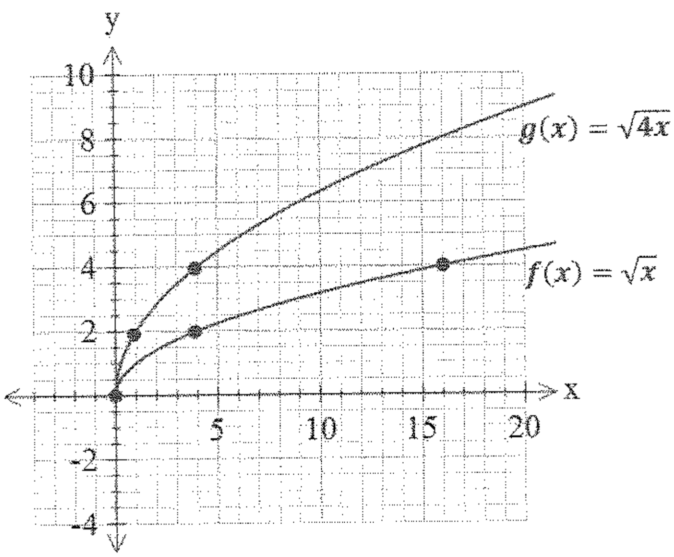
2. Graph the following functions on the same coordinate plane.

$f(x) = \sqrt{x}$

$g(x) = \sqrt{4x}$

I know that the graph of $g$ is a horizontal shrink of the graph of $f$. The 4 will cause the graph to shrink horizontally by a scale factor of $\frac{1}{4}$.




I can graph $y = g(x)$ by multiplying the $x$-value of each coordinate on the graph of $y = f(x)$ by $\frac{1}{4}$.

3. Explain how the graphs of functions $g(x) = 2x^3$ and $h(x) = (2x)^3$ are related.

*The graphs of $y = g(x)$ and $y = h(x)$ are both transformations of the graph of the parent function $f(x) = x^3$. The graph of $y = g(x)$ is a vertical stretch of the graph of $y = f(x)$ while the graph of $y = h(x)$ is a horizontal shrink of the graph of $y = f(x)$. The graph of $y = g(x)$ can be obtained by multiplying the $y$-value of each coordinate on the graph of $y = f(x)$ by 2. The graph of $y = h(x)$ can be obtained by multiplying the $x$-value of each coordinate on the graph of $y = f(x)$ by $\frac{1}{2}$.*

4. Explain how the graphs of functions $j(x) = -x^3$ and $k(x) = (-x)^3$ are related.

*The graphs of $y = j(x)$ and $y = k(x)$ are both transformations of the graph of the parent function $f(x) = x^3$. They would each result in the same graph because $k(x) = (-x)^3$ can be rewritten as follows:*

$$k(x) = (-x)^3 = (-1)^3(x)^3 = -x^3.$$

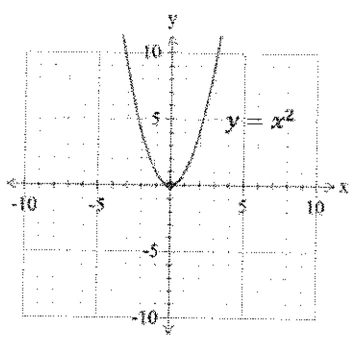
*Both graphs can be obtained by reflecting the graph of $y = f(x)$ across the $x$-axis.*

# Lesson 21: Transformations of the Quadratic Parent Function, $f(x) = x^2$

Sketch the graphs of the functions below based on transformations of the graph of the parent function $f(x) = x^2$.

1. $g(x) = -(x+1)^2 + 8$

Using what I know about transformations of functions, I see that the graph of $y = g(x)$ will be the graph of $f(x) = x^2$ shifted left 1 unit, reflected across the $x$-axis, and shifted up 8 units. I could do the transformations in three separate steps.

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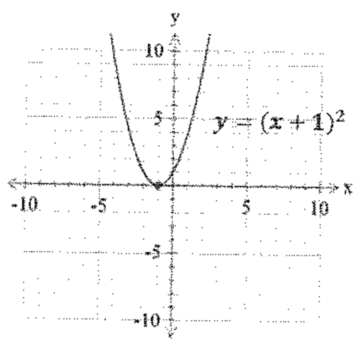


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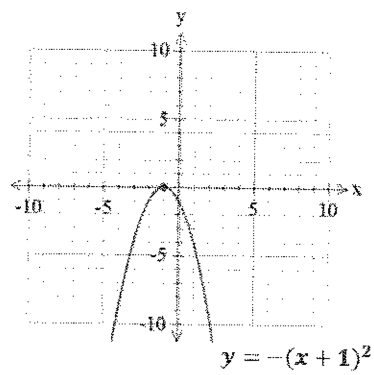


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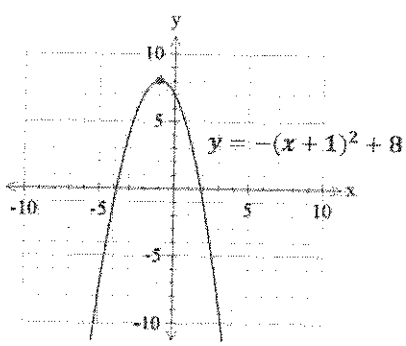




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2. $h(x) = 4x^2 - 16x + 11$

$h(x) = 4(x^2 - 4x + \_\_\_\_) + 11 + \_\_\_\_$

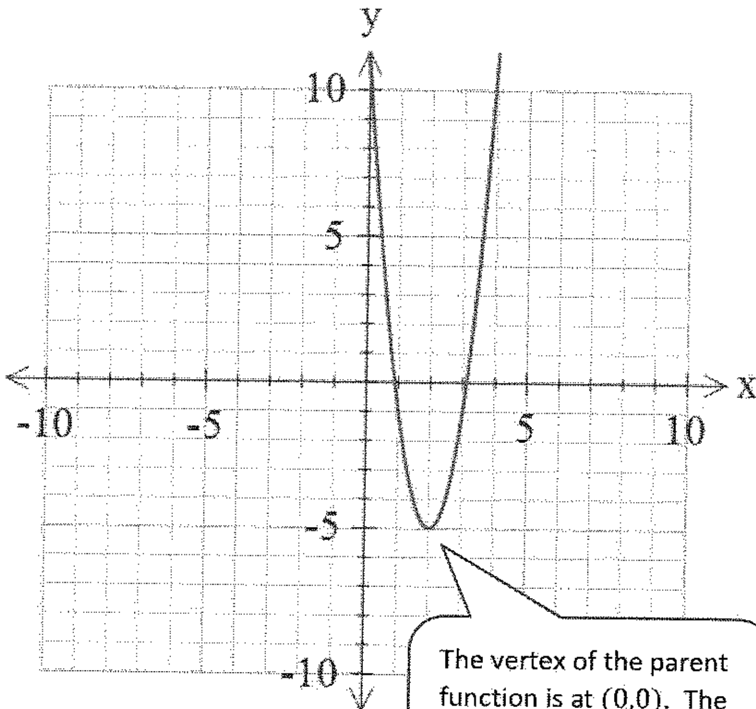
$h(x) = 4(x^2 - 4x + 4) + 11 + 4(-4)$

$h(x) = 4(x^2 - 4x + 4) + 11 - 16$

$h(x) = 4(x - 2)^2 - 5$

In order to identify the transformations, I need to rewrite the function by completing the square.

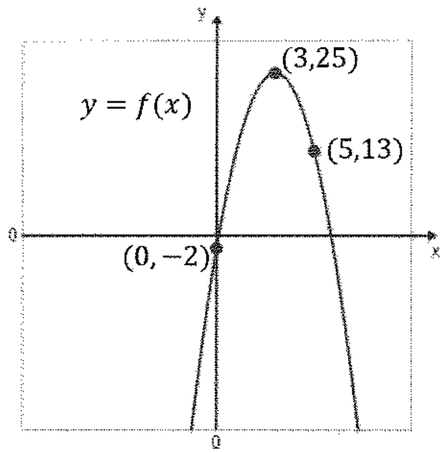
Now I can see the transformations required to graph $y = h(x)$ by starting with the graph of $f(x) = x^2$. The graph will be shifted right 2 units, stretched vertically by a scale factor of 4, and shifted down 5 units.




The vertex of the parent function is at $(0,0)$. The vertex of the translated function is at $(2, -5)$.

# Lesson 22: Comparing Quadratic, Square Root, and Cube Root Functions Represented in Different Ways

1. Consider two quadratic functions, $f$ and $g$. A portion of the graph of $f$ and a formula for $g$ are given below.




$g(x) = -2x^2 + 12x$

I know that the $y$-intercept is the value of $y$ where the graph of the function intersects the $y$-axis.

a. Which function has a larger $y$-intercept?

*The $y$-intercept for $f$ is $-2$.*

*The $y$-intercept for $g$ is $0$.*

$g(0) = -2(0)^2 + 12(0) = 0$

*The function $g$ has a larger $y$-intercept.*

b. Which function has a larger maximum value?

*From the graph, the maximum value of $f$ appears to be $25$.*

*For $g(x) = -2x^2 + 12x$, the axis of symmetry is*

$x = -\frac{(12)}{2(-2)} = 3.$

$g(3) = -2(3)^2 + 12(3) = 18$

*The maximum value of $g$ is $18$.*

*The function $f$ has a larger maximum value.*

I will need to find the vertex of the graph of $g$. Since the function is given in standard form, I could find the axis of symmetry, which is the $x$-coordinate of the vertex, by using the formula $x = -\frac{b}{2a}$.

c. Which function has a larger average rate of change on the interval $1 \leq x \leq 3$?

*The average rate of change for $f$ on the interval $1 \leq x \leq 3$ is* $\frac{f(3)-f(1)}{3-1}$.

$$\frac{f(3)-f(1)}{3-1} = \frac{25-13}{2} = 6$$

To find $f(1)$, I need to use the symmetry of the graph.

*The average rate of change for $g$ on the interval $1 \leq x \leq 3$ is* $\frac{g(3)-g(1)}{3-1}$.

To find $g(3)$, I substitute 3 into the formula for $x$. To find $g(1)$, I substitute 1 into the formula for $x$.

$$\frac{g(3)-g(1)}{3-1} = \frac{18-10}{2} = 4$$

*The function $f$ has a larger average rate of change on the interval $1 \leq x \leq 3$.*

2. The function $f(x) = 6.9\sqrt{x}$ gives the velocity of a car, in miles per hour, for a particular stopping distance $x$, in meters, when a car is traveling on dry asphalt. The function $g$ gives the velocity of a car, in miles per hour, for a particular stopping distance $x$, in meters, when a car is traveling on black ice. Select values of $g(x)$ are given in the table below.

| $x$, in meters | $g(x)$, in mph |
|---|---|
| 0 | 0 |
| 25 | 30 |
| 50 | 40 |
| 150 | 70 |
| 250 | 90 |

a. In order to be able to stop within 25 meters, how much slower should a person drive on ice than on dry asphalt?

$f(25) = 6.9\sqrt{25} = 34.5$

$g(25) = 30$

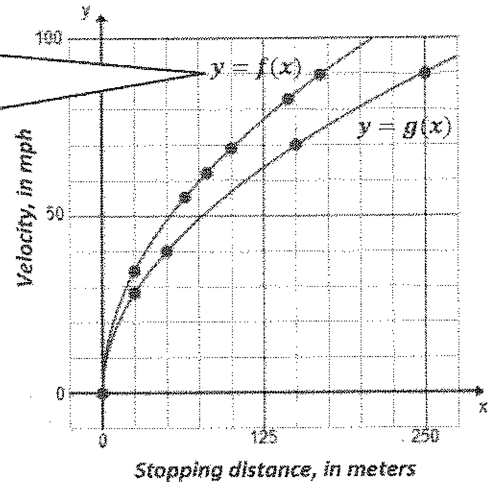
This means that a car traveling at a velocity of 34.5 mph will be able to stop within 25 meters.

*In order to stop within 25 meters, a person should drive 4.5 mph slower on ice than when driving on dry asphalt.*

b. Graph both functions on the same coordinate plane. Suppose a car is traveling at a velocity of $60$ mph. Approximately how much greater is the stopping distance if the car is on black ice rather than dry asphalt?

I can graph $y = f(x)$ by making a table of values. I choose input values that are perfect squares.

I can use the graphs of $f$ and $g$ to approximate the stopping distances.




***At a velocity of $60$ mph, a car traveling on dry asphalt requires a distance of approximately $75$ meters to stop. A car traveling at the same velocity on black ice requires a distance of approximately $110$ meters to stop. It will take the car traveling on ice approximately $35$ meters longer to stop than the car traveling on dry asphalt.***



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# Lesson 23: Modeling with Quadratic Functions

## Modeling Projectile Motion with a Quadratic Function

1. A rocket is shot straight up in the air from a platform that is 8 feet above the ground. The initial velocity of the rocket is 288 feet per second.

   a. Write a formula that models the height of the rocket, $h$, in feet, $t$ seconds after the rocket is launched.

   $h(t) = -16t^2 + 288t + 8$

   Since I am working with units of feet, I know I should use the formula $h(t) = -16t^2 + v_0t + h_0$, where $v_0$ represents the initial velocity, and $h_0$ represents the initial height.

   b. How long does it take for the rocket to reach its maximum height?

   $h(t) = -16t^2 + 288t + 8$

   $h(t) = -16(t^2 - 18t + 81) + 8 - 16(-81)$

   $h(t) = -16(t^2 - 18t + 81) + 8 + 1296$

   $h(t) = -16(t - 9)^2 + 1304$

   I need to find the $t$-value of the vertex. I could rewrite the height function in completed square form to find the vertex.

   I see that the vertex of the graph of the height function is $(9,1304)$.

   *The rocket reaches its maximum height 9 seconds after being launched.*

   c. What is the maximum height of the rocket?

   *The maximum height of the rocket is $1,304$ feet.*

d. How long does it take for the rocket to hit the ground?

$$h(t) = 0$$
$$-16(t-9)^2 + 1304 = 0$$
$$(t-9)^2 = \frac{163}{2}$$
$$t - 9 = \pm\sqrt{\frac{163}{2}}$$
$$t = 9 + \sqrt{\frac{163}{2}} \text{ or } t = 9 - \sqrt{\frac{163}{2}}$$

I need to find the $t$-value for which $h(t) = 0$. I could use the standard form of the function from part (a) or the completed square form that I found in part (b).

*The solutions rounded to the nearest ten thousandth are* $18.0277$ *and* $-0.0277$.

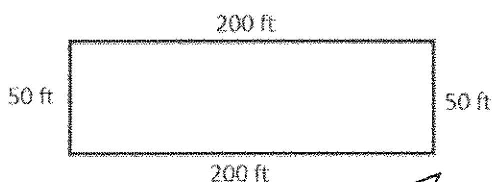
*The rocket will hit the ground after approximately* $18.0277$ *seconds.*

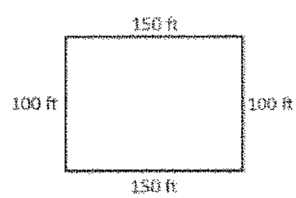
I know that this value has no meaning within the context of this problem.

### Modeling Area with a Quadratic Function

2. A school wishes to construct a rectangular enclosure that will house a playground. The school has 500 feet of fencing available to build the enclosure.

   a. Draw two examples of ways in which the enclosure could be constructed.




I see that there are infinitely many ways in which the enclosure could be constructed, as long as the perimeter is equal to 500 ft.

   b. Let $w$ represent the width of the enclosure. Write an expression to represent the length of the enclosure in terms of $w$.

$$2w + 2l = 500$$
$$w + l = 250$$
$$l = 250 - w$$

I know that the sum of the four sides must equal 500.

*The length of enclosure can be represented with the expression* $(250 - w)$.

c. Write a formula expressing the area of the enclosure, $A$, as a function of its width, $w$.

$A = wl$

$A(w) = w(250 - w)$

Starting with the area formula for a rectangle, I can use my expression from part (b) to write the formula in terms of $w$.

d. If the goal is to maximize the area of the enclosure, what should the dimensions be?

I need to find the $w$-value of the vertex of the area function graph. I can use the factored form of the function from part (c).

$A(w) = w(250 - w) = 0$

$w = 0$ *or* $w = 250$

*The* $w$*-intercepts of the graph are* $0$ *and* $250$.

*The vertex will occur at* $w = 125$.

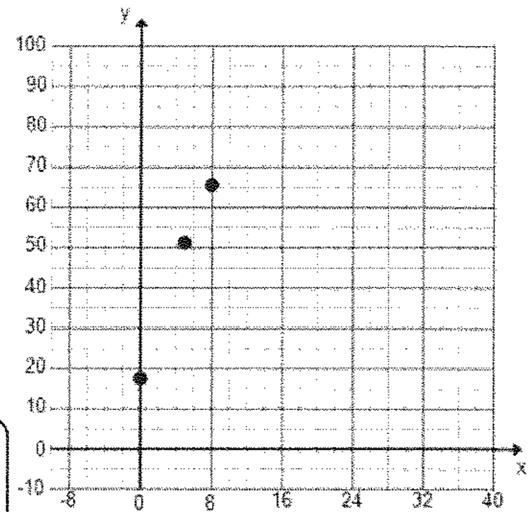
Using the symmetry of the graph, I know that the vertex will occur halfway between the $w$-intercepts.

*In order to maximize the area of the enclosure, the width should be* $125$ *feet, and the length should also be* $125$ *feet.*

I can use my expression from part (b) to find the length.

# Lesson 24: Modeling with Quadratic Functions

Consider a quadratic function whose graph passes through the points $(0,17.5)$, $(5, 51.25)$, and $(8, 65.5)$, shown on the graph below.




1. Write a formula for the quadratic function.

> I know that I am writing a formula in the form $f(x) = ax^2 + bx + c$.
> I need to find the values of the parameters $a$, $b$, and $c$.

$f(0) = a(0)^2 + b(0) + c = 17.5$

$c = 17.5$

> I can use the $y$-intercept to find the value of $c$.

$f(x) = ax^2 + bx + 17.5$

> Now I have two unknown values ($a$ and $b$). I can use the other two points that were given to set up a system of equations.

$f(5) = a(5)^2 + b(5) + 17.5 = 51.25 \rightarrow 25a + 5b = 33.75$

$f(8) = a(8)^2 + b(8) + 17.5 = 65.5 \rightarrow 64a + 8b = 48$

> I am going to solve this system of equations by using the elimination method.

$8(25a + 5b) = 8(33.75) \rightarrow 200a + 40b = 270$

$-5(64a + 8b) = -5(48) \rightarrow -320a - 40b = -240$

$-120a = 30$

$a = -0.25$

> When I add these two equations I get $-120a = 30$.

*Substitute a into one of the original equations:*

$25(-0.25) + 5b = 33.75$

$b = 8$

*The formula for the quadratic function is* $f(x) = -0.25x^2 + 8x + 17.5$.



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2. Graph the quadratic function using the formula found in Problem 1. Show that the graph includes the three points that were given.