G8-M4-Lesson 10: A Critical Look at Proportional Relationships

1. Jurgen types a paper for his Humanities class at a constant speed. He types 12 pages, and it took him 66 minutes.

   a. What fraction represents his constant speed, C?

   $$C = \frac{12}{66} = \frac{2}{11}$$

   To write the fraction for his constant speed, I have to compare the number of pages typed to the interval of time spent typing.

   b. Write the fraction that represents his constant speed, C, if he types y pages in 24 minutes.

   $$C = \frac{y}{24}$$

   c. Write a proportion using the fractions from parts (a) and (b) to determine how many pages he types after 24 minutes. Round your answer to the hundredths place.

   $$\frac{2}{11} = \frac{y}{24}$$
   $$2(24) = 11(y)$$
   $$48 = 11(y)$$
   $$\frac{1}{11}(48) = \frac{1}{11}(11)y$$
   $$4.36 \approx y$$

   Jurgen types approximately 4.36 pages in 24 minutes.

   d. Write a two-variable equation to represent how many pages Jurgen can type over any time interval.

   Let y represent the number of pages typed. Let x represent the number of minutes typed.

   When I write a two-variable equation, I have to remember to define my variables.

   $$\frac{2}{11} = \frac{y}{x}$$
   $$2(x) = 11(y)$$
   $$\frac{1}{11}(2)x = \frac{1}{11}(11)y$$
   $$\frac{2}{11}x = y$$

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2. Parker runs at a constant speed of 6.25 miles per hour.

   a. If he runs for y miles and it takes him x hours, write the two-variable equation to represent the number of miles Parker can run in x hours.

   Let y represent the number of miles run. Let x represent the number of hours run.

$$\frac{6.25}{1} = \frac{y}{x}$$
$$6.25x = y$$

   b. Parker has been training for a marathon by running to the school 11 miles from his house, then to the park 2 miles from the school, and then returning home, which is 14 miles from the park. Assuming he runs at a constant speed the entire time, how long will it take him to get back home after running his route? Round your answer to the hundredths place.

   Total miles: $11 + 2 + 14 = 27$. *Let x be the number of hours run.*

$$6.25x = 27$$
$$\frac{1}{6.25}(6.25)x = \frac{1}{6.25}(27)$$
$$x = 4.32$$

   It will take Parker 4.32 *hours to run* 27 *miles.*

3. Jared walks from baseball practice to his aunt's house, a distance of 6 miles, in 90 minutes. Assuming he walks at a constant speed, C, how far does he walk in 20 minutes? Round your answer to the hundredths place.

   Let y represent the number of miles walked.

   Since $\frac{6}{90} = C$ *and* $\frac{y}{20} = C$, *then*

$$\frac{6}{90} = \frac{y}{20}$$
$$6(20) = 90y$$
$$120 = 90y$$
$$\frac{1}{90}(120) = \frac{1}{90}(90)y$$
$$\frac{120}{90} = y$$
$$1.33 \approx y$$

   Jared walks approximately 1.33 *miles in* 20 *minutes.*

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4. Sammy bikes 3 miles every night for exercise. It takes him exactly 1.75 hours to finish his ride.

a. Assuming he rides at a constant rate, write an equation that represents how many miles, y, Sammy can ride in x hours.

$$\frac{3}{1.75} = \frac{y}{x}$$
$$3x = 1.75y$$
$$\frac{1}{1.75}(3)x = \frac{1}{1.75}(1.75)y$$
$$\frac{3}{1.75}x = y$$

I don't need to define my variables for this problem because they have already done it in the problem.

b. Use your equation from part (a) to complete the table below. Use a calculator, and round all values to the hundredths place.

x (hours)	Linear Equation in y: $\frac{3}{1.75}x = y$	y (miles)
0.25	$\frac{3}{1.75}(0.25) = y$	0.43
0.5	$\frac{3}{1.75}(0.5) = y$	0.86
0.75	$\frac{3}{1.75}(0.75) = y$	1.29
1	$\frac{3}{1.75}(1) = y$	1.71
3	$\frac{3}{1.75}(3) = y$	5.14

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G8-M4-Lesson 11: Constant Rate

1. A bus travels at a constant rate of 40 miles per hour.

   What is the distance, d, in miles, that the bus travels in t hours?

   Let C be the constant rate the bus travels. Then, $\frac{40}{1} = C$, and $\frac{d}{t} = C$; therefore, $\frac{40}{1} = \frac{d}{t}$.

   $$\frac{40}{1} = \frac{d}{t}$$
   $$d = 40t$$

   If I can write two fractions, each equal to the constant rate, C, then I can use the proportional relationship to solve for d.

2. A teenage boy named Harry can consume 8 hot dogs in 1.25 hours. Assume that the young man eats at a constant rate.

   a. How many hot dogs, y, can be consumed by Harry in t hours?

   Let C be the constant rate Harry eats hot dogs. Then, $\frac{8}{1.25} = C$, and $\frac{y}{t} = C$; therefore, $\frac{8}{1.25} = \frac{y}{t}$.

   $$\frac{8}{1.25} = \frac{y}{t}$$
   $$1.25y = 8t$$
   $$\frac{1.25}{1.25}y = \frac{8}{1.25}t$$
   $$y = 6.4t$$

   b. Pretend that he can eat every hour of every day for a week. How many hot dogs would Harry consume?

   24 hours a day for 7 days is a total of 168 hours.

   $$y = 6.4t$$
   $$y = 6.4(168)$$
   $$y = 1{,}075.2$$

   Once I figure out how many hours are in a week, I can use my equation from part (a) to determine the answer.

   Harry would consume about 1,075 hot dogs in one week.

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3. Your cell phone company charges at a constant rate. The company charges $1.00 for 4 minutes of use.

a. Write an equation to represent the number of dollars, d, that will be charged over any time interval, t.

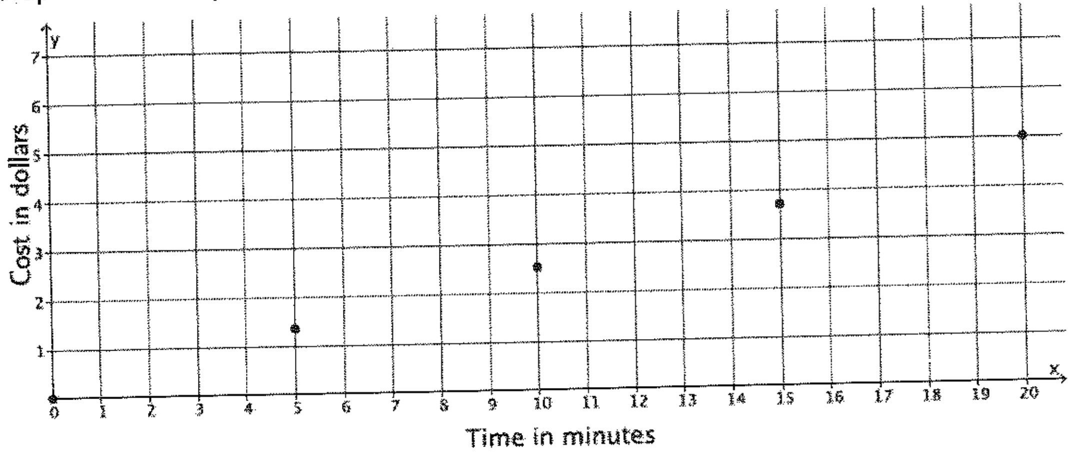
Let C be the constant rate charged per minute. Then, $\frac{1.00}{4} = C$, and $\frac{d}{t} = C$; therefore, $\frac{1.00}{4} = \frac{d}{t}$.

$$\frac{1}{4} = \frac{d}{t}$$
$$4d = 1t$$
$$\frac{1}{4}(4)d = \frac{1}{4}(1)t$$
$$d = 0.25t$$

b. Complete the table below.

t (time in minutes)	Linear Equation: $d = 0.25t$	d (cost in dollars)
0	$d = 0.25(0)$	0
5	$d = 0.25(5)$	1.25
10	$d = 0.25(10)$	2.50
15	$d = 0.25(15)$	3.75
20	$d = 0.25(20)$	5.00

c. Graph the data as points on a coordinate plane.



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d. You used your phone for 18 minutes. About how much will your bill be? Explain.

It will cost between $3.75 *and* $5. *I located* 18 *on the* x*-axis because that is the number of minutes I used. That* x*-value is between the known costs for* 15 *minutes and* 20 *minutes. So my bill will probably be closer to* $5 *because* 18 *is closer to* 20 *than to* 15.

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G8-M4-Lesson 12: Linear Equations in Two Variables

1. Consider the linear equation $x - \frac{2}{5}y = 4$.

   a. Will you choose to fix values for x or y? Explain.

   If I fix values for y, it will make the computations easier. Solving for x can be done in one step.

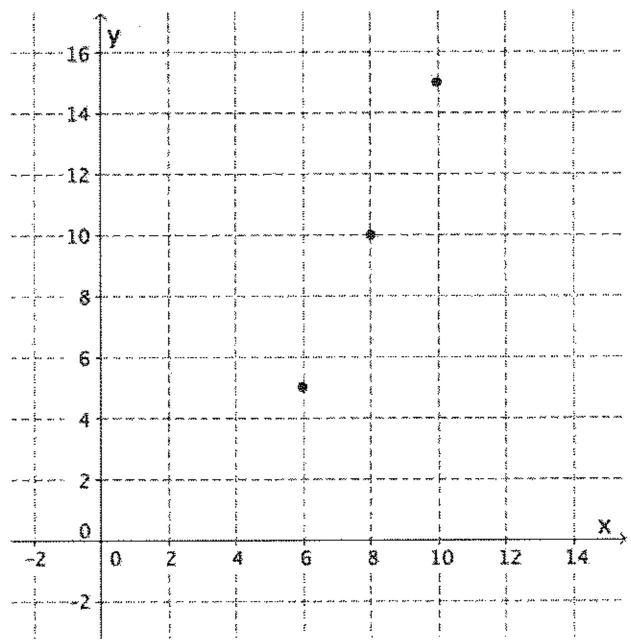
   b. Are there specific numbers that would make your computational work easier? Explain.

   Values for y that are multiples of 5 will make the computations easier. When I multiply $\frac{2}{5}$ by a multiple of 5, I will get a whole number.

   c. Find three solutions to the linear equation $x - \frac{2}{5}y = 4$, and plot the solutions as points on a coordinate plane.

   I'll use the numbers 5, 10, and 15 for y in my table. Once substituted into the equation, I'll get the values for x. Then each pair of x and y will be a point on my graph.

x	Linear Equation: $x - \frac{2}{5}y = 4$	y
6	$x - \frac{2}{5}(5) = 4$ $x - 2 = 4$ $x = 6$	5
8	$x - \frac{2}{5}(10) = 4$ $x - 4 = 4$ $x = 8$	10
10	$x - \frac{2}{5}(15) = 4$ $x - 6 = 4$ $x = 10$	15



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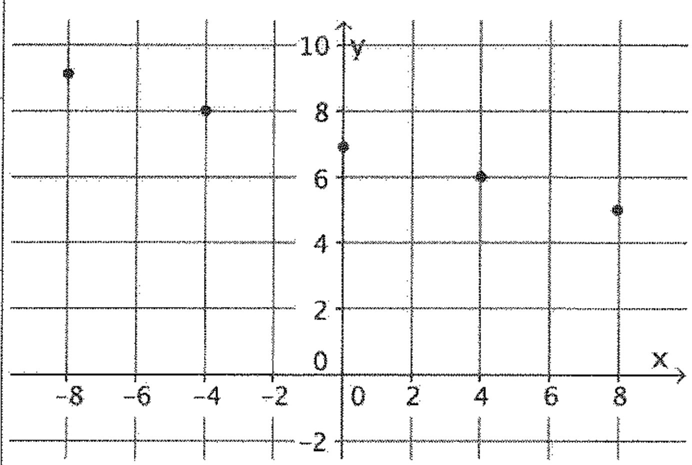
G8-M4-Lesson 13: The Graph of a Linear Equation in Two Variables

1. Find at least five solutions to the linear equation $\frac{1}{4}x + y = 7$, and plot the points on a coordinate plane. What shape is the graph of the linear equation taking?

I should choose values for x that are multiples of 4. I should also be sure to select some positive values for x as well as negative.

x	$\frac{1}{4}x + y = 7$	y
-8	$\frac{1}{4}(-8) + y = 7$ $-2 + y = 7$ $-2 + 2 + y = 7 + 2$ $y = 9$	9
-4	$\frac{1}{4}(-4) + y = 7$ $-1 + y = 7$ $-1 + 1 + y = 7 + 1$ $y = 8$	8
0	$\frac{1}{4}(0) + y = 7$ $0 + y = 7$ $y = 7$	7
4	$\frac{1}{4}(4) + y = 7$ $1 + y = 7$ $1 - 1 + y = 7 - 1$ $y = 6$	6
8	$\frac{1}{4}(8) + y = 7$ $2 + y = 7$ $2 - 2 + y = 7 - 2$ $y = 5$	5



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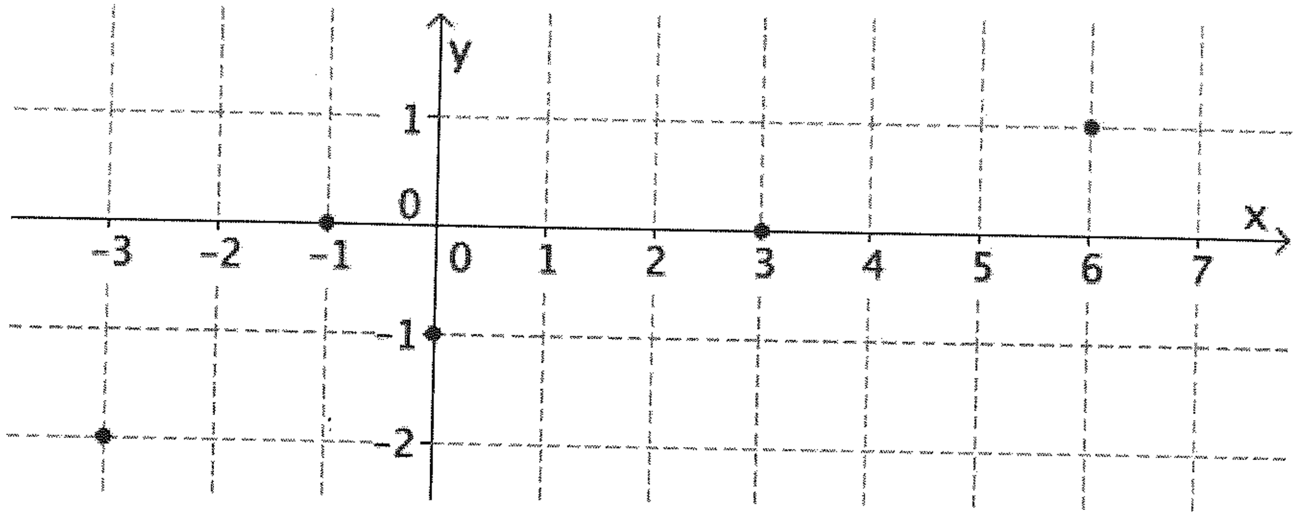
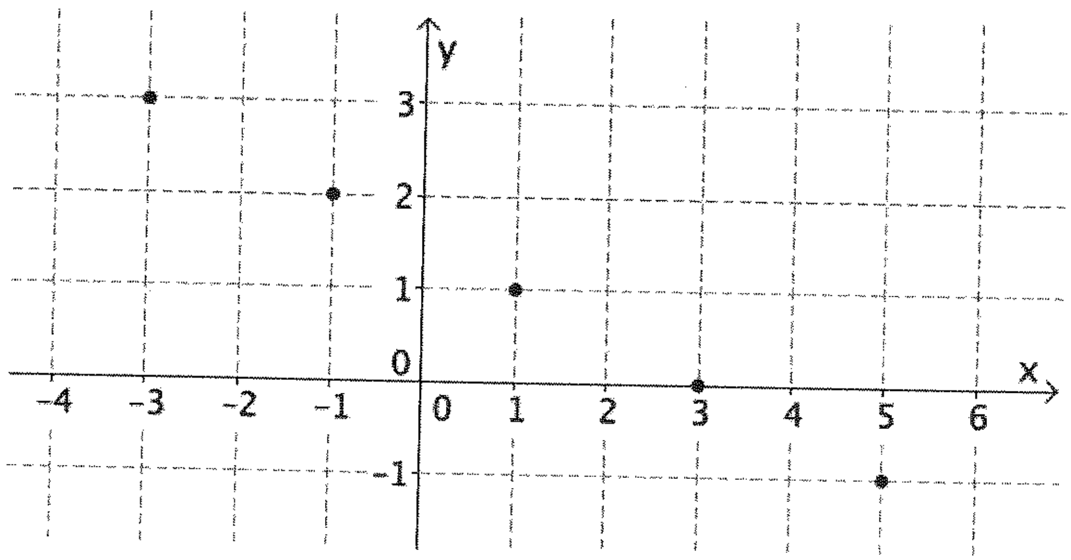
2. Can the following points be on the graph of the equation $x - 3y = 3$? Explain



The graph shown contains the point $(-1, 0)$. If $(-1, 0)$ is on the graph of the linear equation, then it will be a solution to the equation. It is not; therefore, the point cannot be on the graph of the equation, which means the graph shown cannot be the graph of the equation $x - 3y = 3$.

3. Can the following points be on the graph of the equation $2x + 4y = 6$? Explain



Yes, this graph is of the equation $2x + 4y = 6$ because each point on the graph represents a solution to the linear equation $2x + 4y = 6$.

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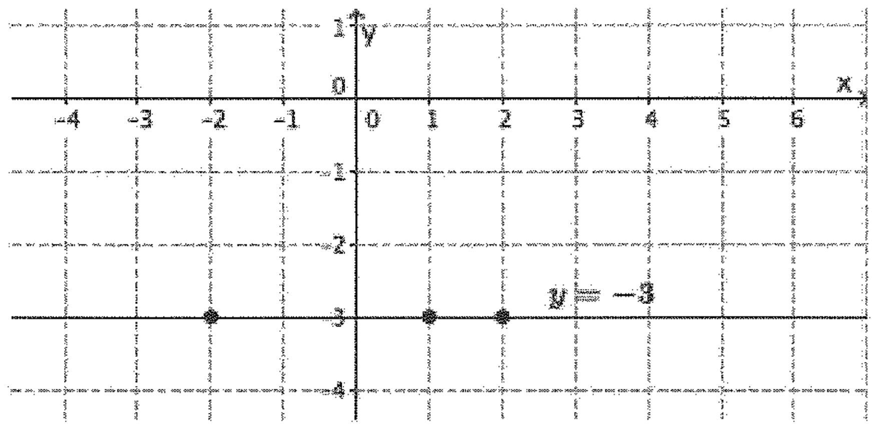
G8-M4-Lesson 14: Graph of a Linear Equation—Horizontal and Vertical Lines

1. Graph the two-variable linear equation $ax + by = c$, where $a = 0$, $b = -2$, and $c = 6$.

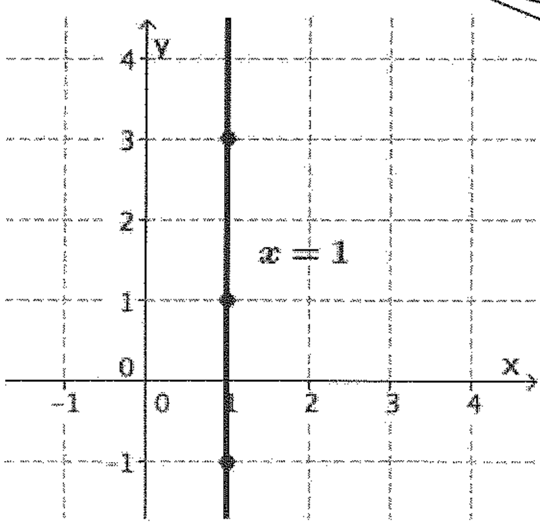
$$ax + by = c$$
$$0x + (-2)y = 6$$
$$-2y = 6$$
$$y = -3$$

I'm not sure how to graph this, so I'll find some solutions using a table like in the last lesson.



2. Graph the linear equation $x = 1$.

I know that this will either be a horizontal or vertical line. Since the equation is $x = 1$, that means that no matter what value I choose for y, the x-value will always be one.



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3. Explain why the graph of a linear equation in the form of $x = c$ is the vertical line, parallel to the y-axis passing through the point $(c, 0)$.

   The graph of $x = c$ passes through the point $(c, 0)$, which means the graph of $x = c$ cannot be parallel to the x-axis because the graph intersects it. For that reason, the graph of $x = c$ must be a vertical line parallel to the y-axis.

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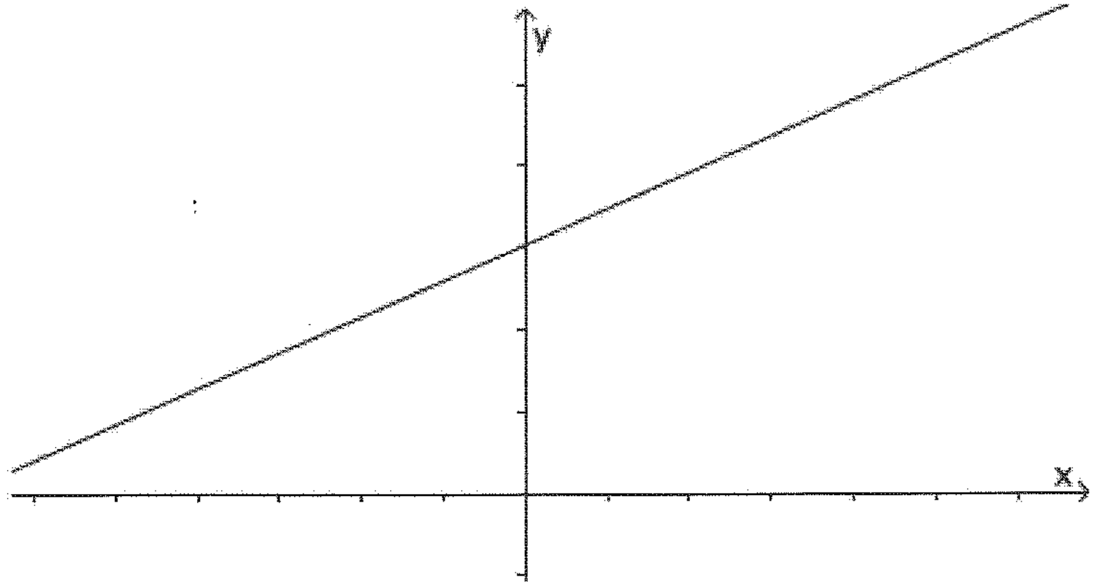
G8-M4-Lesson 15: The Slope of a Non-Vertical Line

1. Does the graph of the line shown below have a positive or negative slope? Explain



The graph of this line has a positive slope. It is left-to-right inclining, which is an indication of positive slope.

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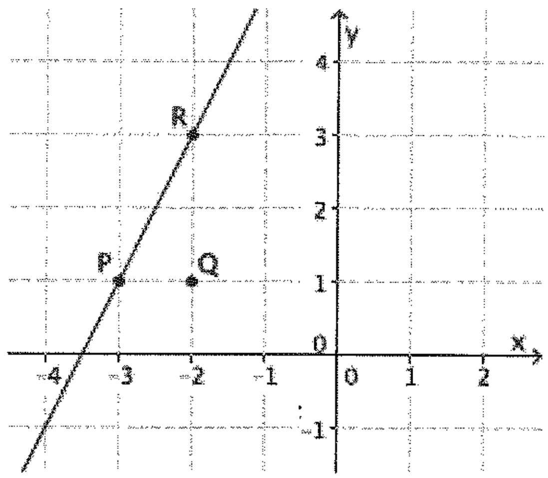
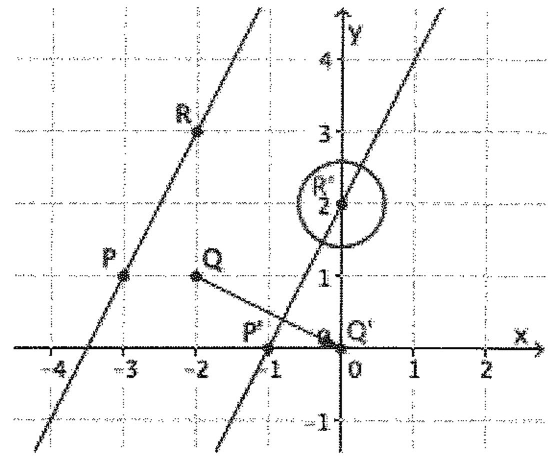
2. What is the slope of this non-vertical line? Use your transparency if needed.





The slope of this line is 2, *so* $m = 2$.

Since the distance between points P and Q is 1 unit, I can trace everything onto a transparency and map point Q to the origin. The location of the translated point R gives me the slope of the line.

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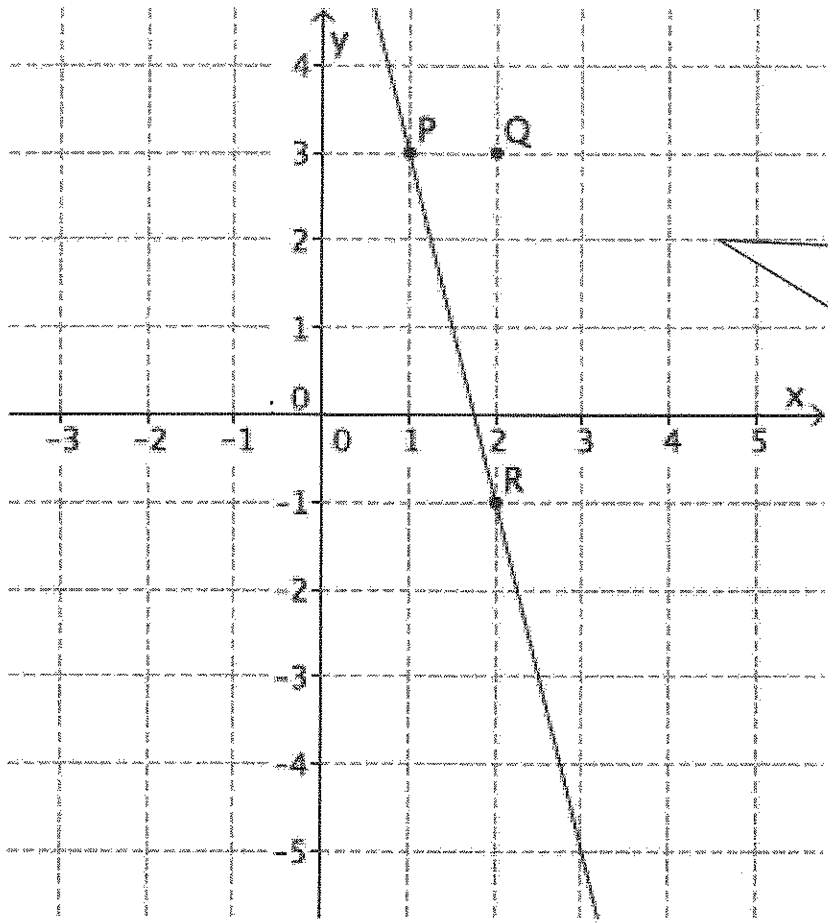
3. What is the slope of this non-vertical line? Use your transparency if needed.



I can tell by the line that the slope will be negative. Just like I did in the last problem, I will use my transparency and translation to figure out the number that represents the slope.

The slope of this line is -4, *so* $m = -4$.

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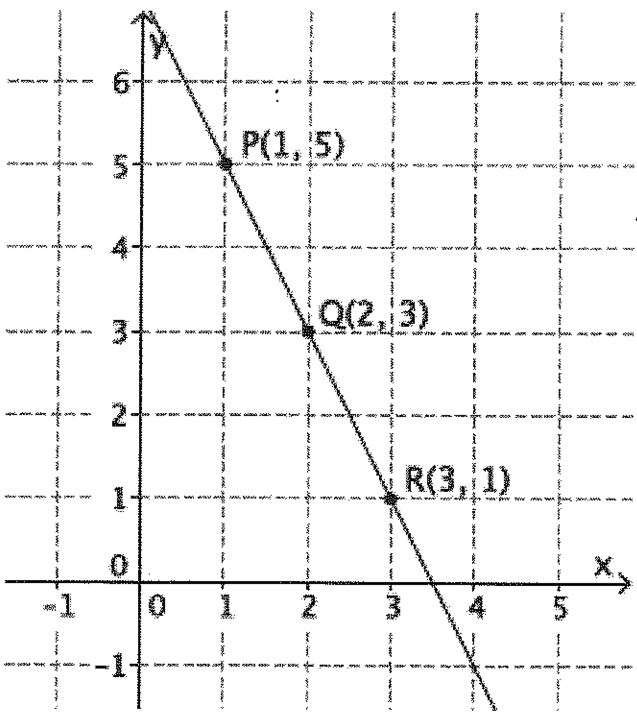
G8-M4-Lesson 16: The Computation of the Slope of a Non-Vertical Line

1. Calculate the slope of the line using two different pairs of points.



I need to choose three points on the line. The points $P(p_1, p_2)$ and $Q(q_1, q_2)$ are used in my first slope equation. I need to remember that no matter how the slope is written, the difference in the 2^{nd} values (y-values) is in the numerator of the slope, and the difference in the 1^{st} values (x-values) is in the denominator of the slope. The equation $m = \frac{q_2 - p_2}{p_1 - q_1}$ would be wrong since the values of point Q do not come first in each difference.

$$m = \frac{p_2 - q_2}{p_1 - q_1} = \frac{5-3}{1-2} = \frac{2}{-1} = -2$$

$$m = \frac{r_2 - q_2}{r_1 - q_1} = \frac{1-3}{3-2} = \frac{-2}{1} = -2$$

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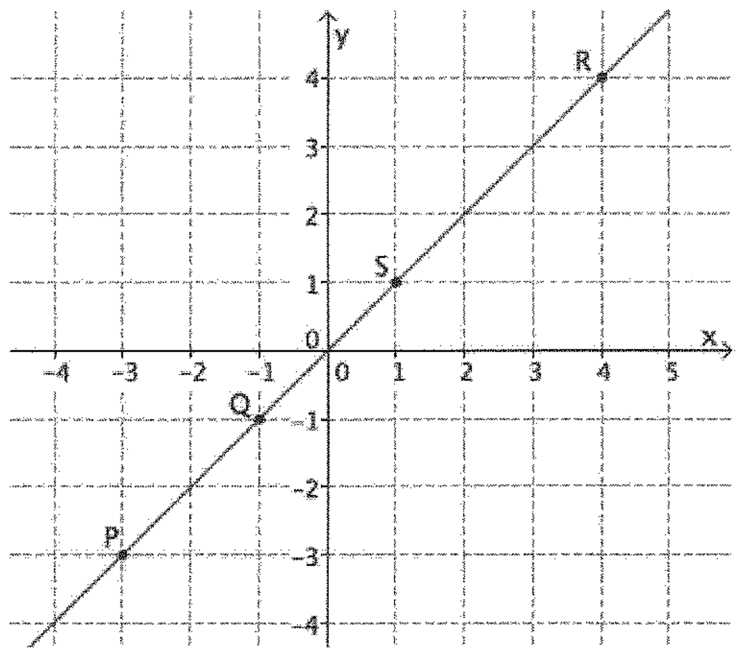
2. Calculate the slope of the line using two different pairs of points.

   a. Select any two points on the line to compute the slope.

   Let the two points be $P(-3,-3)$ *and* $Q(-1,-1)$.

   $$m = \frac{p_2 - q_2}{p_1 - q_1} = \frac{-3-(-1)}{-3-(-1)} = \frac{-2}{-2} = 1$$



   b. Select two different points on the line to calculate the slope.

   Let the two points be $S(1,1)$ *and* $R(4,4)$.

   $$m = \frac{s_2 - r_2}{s_1 - r_1} = \frac{1-4}{1-4} = \frac{-3}{-3} = 1$$

   c. What do you notice about your answers in parts (a) and (b)? Explain.

   The slopes are equal in parts (a) and (b). This is true because of what we know about similar triangles. The slope triangle that is drawn between the two points selected in part (a) is similar to the slope triangle that is drawn between the two points in part (b) by the AA criterion. Then, because the corresponding sides of similar triangles are equal in ratio, the slopes are equal.

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3. Your teacher tells you that a line goes through the points $\left(1, \frac{3}{4}\right)$ and $(-2, -3)$.

   a. Calculate the slope of this line.

$$m = \frac{p_2 - r_2}{p_1 - r_1}$$
$$= \frac{\frac{3}{4} - (-3)}{1 - (-2)}$$
$$= \frac{3\frac{3}{4}}{3}$$
$$= \frac{5}{4}$$

   b. Do you think the slope will be the same if the order of the points is reversed? Verify by calculating the slope, and explain your result.

   The slope should be the same because we are joining the same two points. Since the slope of a line can be computed using any two points on the same line, it makes sense that it does not matter which point we name as P and which point we name as R.

$$m = \frac{r_2 - p_2}{r_1 - p_1}$$
$$= \frac{-3 - \frac{3}{4}}{-2 - 1}$$
$$= \frac{-3\frac{3}{4}}{-3}$$
$$= \frac{5}{4}$$

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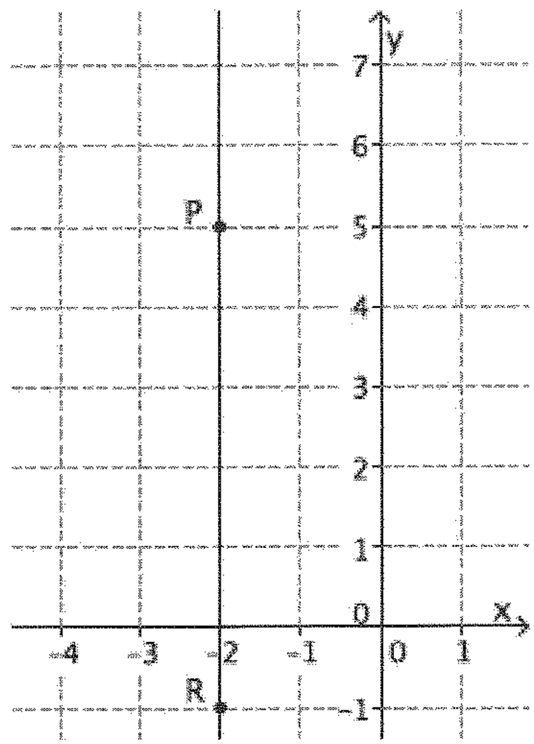
4. Each of the lines in the lesson was non-vertical. Consider the slope of a vertical line, $x = -2$. Select two points on the line to calculate slope. Based on your answer, why do you think the topic of slope focuses only on non-vertical lines?

$$m = \frac{r_2 - p_2}{r_1 - p_1}$$
$$= \frac{-1 - 5}{-2 - (-2)}$$
$$= \frac{-6}{0}$$

The computation of slope using the formula leads to a fraction with zero as its denominator, which is undefined. The topic of slope does not focus on vertical lines because the slope of a vertical line is undefined.



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G8-M4-Lesson 17: The Line Joining Two Distinct Points of the Graph $y = mx+b$ Has Slope m

1. Solve the following equation for y: $-3x + 9y = 18$. Then, answer the questions that follow.

$$-3x + 9y = 18$$
$$-3x + 3x + 9y = 18 + 3x$$
$$9y = 18 + 3x$$
$$\frac{9}{9}y = \frac{18}{9} + \frac{3}{9}x$$
$$y = 2 + \frac{1}{3}x$$
$$y = \frac{1}{3}x + 2$$

   a. Based on your transformed equation, what is the slope of the linear equation $-3x + 9y = 18$?

   The slope is $\frac{1}{3}$.

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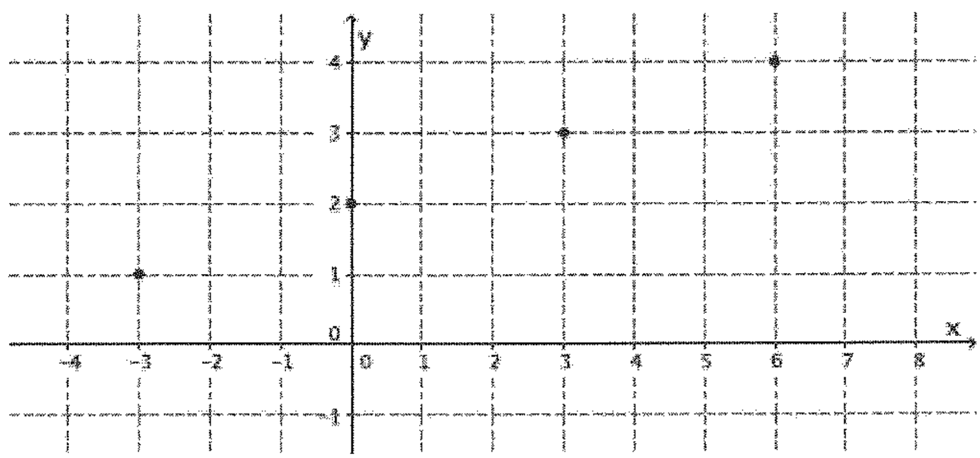
b. Complete the table to find solutions to the linear equation.

Since the slope is a fraction, $\frac{1}{3}$, I need to choose x-values that are multiples of 3.

x	Transformed Equation: $y = \frac{1}{3}x + 2$	y
-3	$y = \frac{1}{3}(-3) + 2$ $= -1 + 2$ $= 1$	1
0	$y = \frac{1}{3}(0) + 2$ $= 2$	2
3	$y = \frac{1}{3}(3) + 2$ $= 1 + 2$ $= 3$	3
6	$y = \frac{1}{3}(6) + 2$ $= 2 + 2$ $= 4$	4

c. Graph the points on the coordinate plane.



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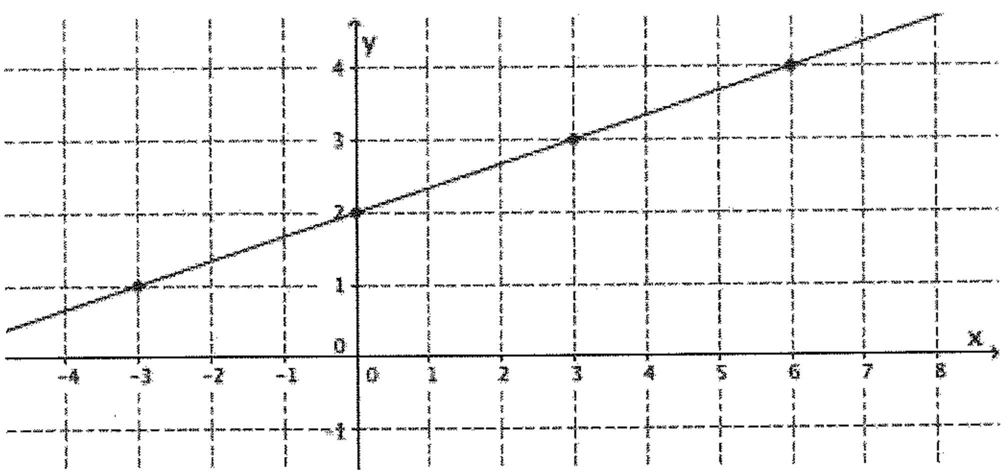
d. Find the slope between any two points.

Using the points $(-3, 1)$ *and* $(3, 3)$,

$$m = \frac{1-3}{-3-3}$$
$$= \frac{-2}{-6}$$
$$= \frac{1}{3}$$

e. The slope you found in part (d) should be equal to the slope you noted in part (a). If so, connect the points to make the line that is the graph of an equation of the form $y = mx + b$ that has slope m.



f. Note the location (ordered pair) that describes where the line intersects the y-axis.

$(0, 2)$ *is the location where the line intersects the* y*-axis.*

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G8-M4-Lesson 18: There Is Only One Line Passing Through a Given Point with a Given Slope

Graph each equation on a separate pair of x- and y-axes. Students need graph paper to complete the Problem Set.

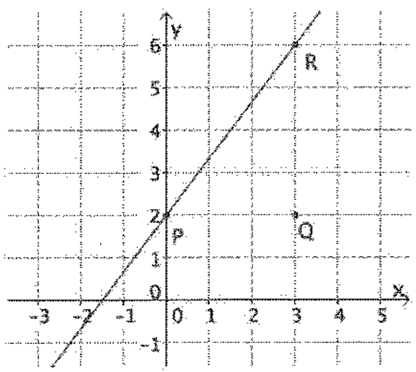
1. Graph the equation $y = \frac{4}{3}x + 2$.

> I know the equation is in slope-intercept form, $y = mx + b$, the number m represents the slope of the graph, and the point $(0, b)$ is the location where the graph of the line intersects the y-axis.

   a. Name the slope and the y-intercept point.

   The slope is $m = \frac{4}{3}$, and the y-intercept point is $(0, 2)$.

   b. Graph the known point, and then use the slope to find a second point before drawing the line.



> I know that $m = \frac{|QR|}{|PQ|}$. Since $|PQ| = 3$, I need to go 3 units from the right of point P, the y-intercept, to find point Q. Since $|QR| = 4$ and the slope is positive, I need to go up 4 units from point Q to find point R.

2. Graph the equation $y = -\frac{1}{3}x + 5$.

   a. Name the slope and the y-intercept point.

   The slope is $m = -\frac{1}{3}$, and the y-intercept point is $(0, 5)$.

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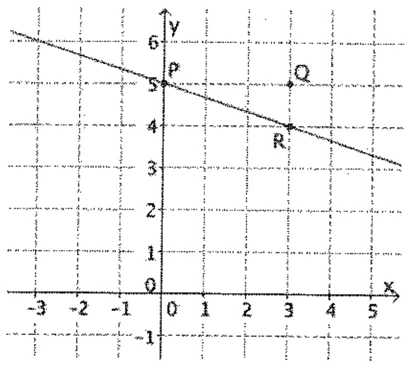
b. Graph the known point, and then use the slope to find a second point before drawing the line.



I need to go three units to the right of point P and mark point Q. Since the slope is negative, I need to go down 1 unit from point Q to find point R.

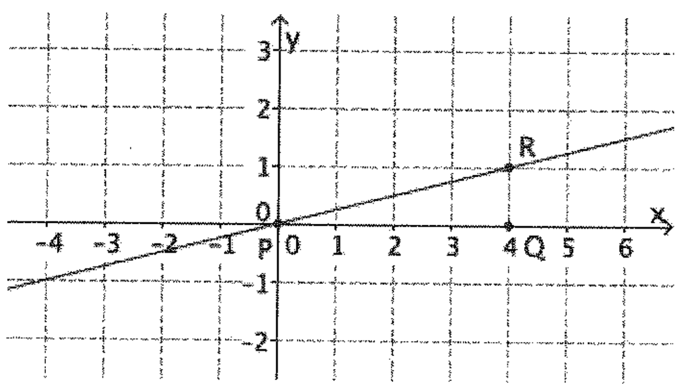
3. Graph the equation $y = \frac{1}{4}x$.

Rewriting the equation in slope-intercept form, $y = \frac{1}{4}x + 0$, helps me to see the y-intercept point is $(0,0)$.

a. Name the slope and the y-intercept point.

The slope is $m = \frac{1}{4}$, *and the y-intercept point is* $(0, 0)$.

b. Graph the known point, and then use the slope to find a second point before drawing the line.



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4. Graph the equation $2x + 2y = 2$.

I need to rewrite the equation in slope-intercept form to help me name the slope and y-intercept point more easily.

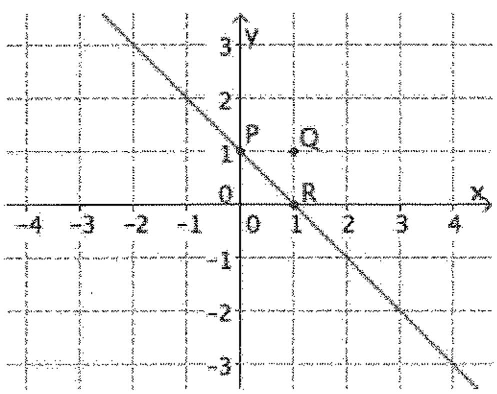
a. Name the slope and the y-intercept point.

$$2x + 2y = 2$$
$$2x - 2x + 2y = -2x + 2$$
$$2y = -2x + 2$$
$$\frac{2}{2}y = -\frac{2}{2}x + \frac{2}{2}$$
$$y = -x + 1$$

The slope -1 is equivalent to the fraction $-\frac{1}{1}$.

The slope is $m = -1$, and the y-intercept point is $(0, 1)$.

b. Graph the known point, and then use the slope to find a second point before drawing the line.



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