Lesson 1: Multiplying and Factoring Polynomial Expressions

1. Factor out the greatest common factor.
   $36x^2y^2 - 15x^3y^4 + 27y^5$
   $3y^2(12x^2 - 5x^3y^2 + 9y^3)$

I use a chart to organize my terms like we did in class. The shaded columns have the same factor in each term.

$36x^2y^2$	2	2	3	3	x	x		y	y			
$-15x^3y^4$	-1	5	3		x	x	x	y	y	y	y	
$27y^5$	3	3	3					y	y	y	y	y
			3					y	y			

$12x^2$
$-5x^3y^2$
$9y^3$

→ GCF: $3y^2$

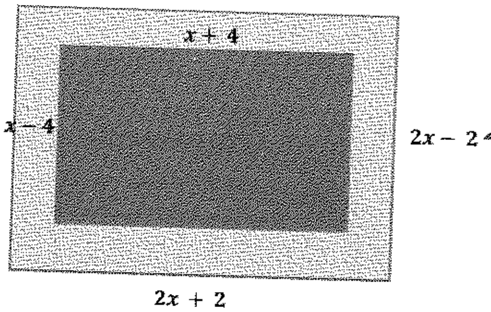
2. Multiply.
   $(k+5)^2$

$$\begin{aligned}(k+5)^2 &= (k+5)(k+5)\\ &= k^2 + 5k + 5k + 25\\ &= k^2 + 10k + 25\end{aligned}$$

I notice this is the square of a binomial; I need to expand the expression and apply the distributive property.

I can also use $(a+b)^2 = a^2 + 2ab + b^2$ and the patterns in this equation to rewrite the expression.

3. A backyard swimming pool is shown in the diagram below. It is surrounded by a paved patio. The pool and the patio take up the entire back yard. The width of the inside of the pool is represented by $x - 4$ and the length by $x + 4$. The width of the entire yard is represented by $2x - 2$ and the length by $2x + 2$.



These expressions represent the length and width of the pool and the surrounding patio.

a. Write an expression to represent the area of the entire yard (larger rectangle).

$(2x - 2)(2x + 2)$ *or* $4x^2 - 4$

Since $a = 2x$, *and* $b = 2$,

the product can be rewritten as $(2x)^2 - (2)^2$,

which is equal to $4x^2 - 4$.

I use the difference of squares to rewrite the product.

$$a^2 - b^2 = (a + b)(a - b)$$

b. Write an expression to represent the area of the pool (inner rectangle).

$(x - 4)(x + 4)$ *or* $x^2 - 16$

c. Express the area of the surrounding patio as a polynomial in terms of x. (Hint: You will have to add or subtract polynomials to get your final answer.)

Subtract the pool's area from the entire yard's area:

$$(4x^2 - 4) - (x^2 - 16) = 4x^2 - 4 - x^2 + 16 = 3x^2 + 12$$

Since we are subtracting the second expression, I remember to add the opposite of both terms when I remove the parentheses.

Lesson 2: Multiplying and Factoring Polynomial Expressions

Factoring Trinomials

1. Factor the trinomial $x^2 - x - 90$ as the product of two binomials, and check your answer by multiplying.

   The first terms in each binomial must multiply to x^2, so use x and x. The last terms in each binomial must multiply to -90, so use -10 and 9.

   $$(x - 10)(x + 9)$$

   Check to see if $(x - 10)(x + 9)$ can be rewritten as $x^2 - x - 90$ using the distributive property.

   $$\begin{aligned}(x-10)(x+9) &= (x-10)(x) + (x-10)(9)\\ &= x^2 - 10x + 9x - 90\\ &= x^2 - x - 90\end{aligned}$$

2. Factor completely, and check your answer by multiplying.

   a. $5p^2 - 30p + 40$

   $5(p^2 - 6p + 8) = 5(p - 4)(p - 2)$

   I factor out a 5, the GCF, from each term. Then I factor the remaining quadratic expression as we did in class.

   Check: $$\begin{aligned}5(p-4)(p-2) &= 5(p^2 - 2p - 4p + 8)\\ &= 5(p^2 - 6p + 8)\\ &= 5p^2 - 30p + 40\end{aligned}$$

   b. $98x^2 - 200$

   $2(49x^2 - 100) = 2(7x - 10)(7x + 10)$

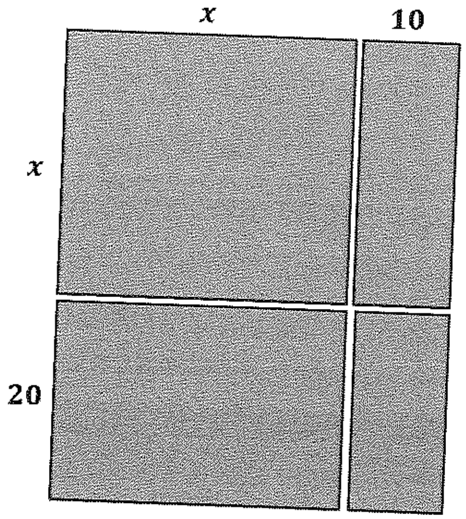
   I know the GCF is 2. The remaining binomial is the difference of two squares, so I can use $a^2 - b^2 = (a + b)(a - b)$ to factor it.

   Check: $$\begin{aligned}2(7x-10)(7x+10) &= 2(49x^2 + 70x - 70x - 100)\\ &= 98x^2 + 140x - 140x - 200\\ &= 98x^2 - 200\end{aligned}$$

Factoring in a Real-World Application

3. The square parking lot is going to be enlarged into a rectangle so that there will be an additional 10 ft. of parking space in the front of the lot and an additional 20 ft. of parking space on the side of the lot, as shown in the figure below. Write an expression in terms of x that can be used to represent the area of the new parking lot.

The area of a rectangle is given by the formula, $A = \text{length} \times \text{width}$*, so the area can be represented as* $(x + 10)(x + 20)$ *or* $x^2 + 30x + 200$.



I check my work by finding the area of each rectangle. The area of the square is x^2. The other rectangular areas are $10x$, $20x$, and 200.

Lesson 3: Advanced Factoring Strategies for Quadratic Expressions

1. Factor the quadratic expression $9x^2 + 66x + 21$.

   $3(3x^2 + 22x + 7) = 3(3x + 1)(x + 7)$

The last terms in each binomial must multiply to 7. I can use $+1$ and $+7$ or $+7$ and $+1$. I can check by multiplying to see if the middle term is $22x$.

I can factor out a 3 first. Then, $3x^2$ must factor into $3x$ and x since 3 is prime. These are the first terms in each binomial.

2. Factor each expression completely.

   a. $100p^2 + 180p + 81$

   Since all terms are positive, use only positive numbers.

   Factor pairs of 100*:* $1 \cdot 100, 2 \cdot 50, 4 \cdot 25, 10 \cdot 10$

   Factor pairs of 81*:* $1 \cdot 81, 3 \cdot 27, 9 \cdot 9$

I need to consider combinations of the factors of the first and last term so that when I multiply, I get the original expression.

$$100p^2 + 180p + 81 = (\_p + \_)(\_p + \_)$$

Try 4 *and* 25 *and* 3 *and* 27*.*

$$(4p + 3)(25p + 27) = 100p^2 + 75p + 108p + 81 = 100p^2 + 183p + 81$$

This is not the correct factorization.

Try 10 *and* 10 *and* 9 *and* 9*.*

$$(10p + 9)(10p + 9) = 100p^2 + 90p + 90p + 81 = 100p^2 + 180p + 81$$

This is the correct factorization.

$$\begin{aligned} 100p^2 + 180p + 81 &= (10p + 9)(10p + 9) \\ &= (10p + 9)^2 \end{aligned}$$

I can rewrite this using exponents since the factors are equal.

b. $-6x^2 - 25x - 25$

Factor pairs of 6: $1 \cdot 6, 2 \cdot 3$

Factor pairs of 25: $1 \cdot 25, 5 \cdot 5$

Using 2 *and* 3 *and* 5 *and* 5: $2(5) + 3(5) = 25$

$$
\begin{aligned}
-(6x^2 + 25x + 25) &= -(\_x + \_)(\_x + \_) \\
&= -(2x + 5)(3x + 5)
\end{aligned}
$$

I factor out a -1 first to make the problem easier. I need to arrange my factors of 6 and 25 so that when I multiply and sum them, they equal 25.

Check by multiplying:

$$-(2x+5)(3x+5) = -(6x^2 + 15x + 10x + 25) = -6x^2 - 25x - 25$$

Lesson 4: Advanced Factoring Strategies for Quadratic Expressions

1. Factor completely.

   a. $16x^2 - 324$

   $$16x^2 - 324 = 4(4x^2 - 81) \\ = 4(2x + 9)(2x - 9)$$

   First, I factor out the common factor 4. Then, I use the difference of two squares where $a = 2x$ and $b = 9$.

   b. $6x^2 + 37x + 6$

   The product of $(a)(c)$ *is* $(6)(6) = 36$. *The factors of* 36 *are* $(1, 36)$, $(-1, -36)$, $(-6, -6)$, $(6, 6)$, $(-3, -12)$, $(3, 12)$, $(4, 9)$, $(-4, -9)$, $(2, 18)$, $(-2, -18)$.

   I can use a table and put the first term in the upper left and the last term in the lower right.

   The factor pair of 36 that adds up to 37 is 1 and 36. This gives me the terms of the other diagonal.

	$1x$	$+6$
$6x$	$6x^2$	$36x$
$+1$	$1x$	6

   Now I record the GCF of each row and write the other factors along the top.

   The factorization of $6x^2 + 37x + 6$ *is* $(6x + 1)(x + 6)$.

   c. The area of the rectangle is represented by the expression $16b^2 + 60b - 100$ square units. Rewrite the expression to show the dimensions in terms of b.

   The GCF is 4. *The product of* $(a)(c)$ *is* $(4)(-25) = -100$.

   $$4(4b^2 + 15b - 25) = 4(4b^2 + 20b - 5b - 25) \\ = 4[(4b^2 + 20b) - (5b + 25)] \\ = 4[4b(b + 5) - 5(b + 5)] \\ = 4(4b - 5)(b + 5)$$

   I need two numbers that multiply to -100 and add to $+15$. 20 and -5 work.

   The 4 *needs to be included in one of the dimensions. Possible dimensions would be* $16b - 20$ *by* $b + 5$. *The dimensions* $4b - 5$ *by* $4b + 20$ *would also work.*

Lesson 5: The Zero Product Property

Solve the following equations.

1. $x^2 + x - 20 = 0$

   $(x+5)(x-4) = 0$

   $x + 5 = 0$ *or* $x - 4 = 0$

   $x = -5$ *or* $x = 4$

   Rewrite this sum as a product. The original equation is true if either factor is equal to 0, which results in a compound equation.

   The solution set is $\{-5, 4\}$.

2. $(3x-3)(4x+2) = 0$

   $3x - 3 = 0$ *or* $4x + 2 = 0$

   $x = 1$ *or* $x = -\frac{1}{2}$

   I set each factor equal to 0 and solve for x.

   The solution set is $\left\{-\frac{1}{2}, 1\right\}$.

3. $t^2 - t = 0$

   $t(t-10) = 0$

   $t = 0$ *or* $t - 10 = 0$

   $t = 0$ *or* $t = 10$

   The solution set is $\{0, 10\}$.

4. $5x^2 - 44x + 120 = -30 + 11x$

   $5x^2 - 55x + 150 = 0$

   $5(x^2 - 11x + 30) = 0$

   $5(x-5)(x-6) = 0$

   $x - 5 = 0$ *or* $x - 6 = 0$

   $x = 5$ *or* $x = 6$

   I need to add 30 to and subtract $11x$ from both sides of the equal sign and then factor out the GCF, which is 5 from each term.

   The solution set is $\{5, 6\}$.

5. $a^2 - 36 = (a+6)(a-2)$

$$(a+6)(a-6) = (a+6)(a-2)$$
$$(a+6)(a-6) - (a+6)(a-2) = 0$$
$$(a+6)\big(a-6-(a-2)\big) = 0$$
$$(a+6)(-4) = 0$$
$$a+6 = 0$$
$$a = -6$$

I cannot divide by $(a+6)$ without changing the solution set. I see that the GCF is $(a+6)$ when all terms are on the same side of the equation.

The solution is -6.

Lesson 6: Solving Basic One-Variable Quadratic Equations

Solve Quadratic Equations

1. $4x^2 - 10 = 14$

$4x^2 - 10 + 10 = 14 + 10$

$4x^2 = 24$

$\frac{4x^2}{4} = \frac{24}{4}$

$x^2 = 6$

$x = \sqrt{6} \textit{ or } x = -\sqrt{6}$

Every perfect square of a non-zero positive number has two square roots, so this equation has two solutions.

The solution set is $\{\sqrt{6}, -\sqrt{6}\}$.

2. $(d+2)^2 = 8$

$\sqrt{(d+2)^2} = \sqrt{8}$

I take the square root of both sides of the equation because the inverse operation of squaring a number is taking the square root.

$d + 2 = \sqrt{8} \textit{ or } d + 2 = -\sqrt{8}$

$d = -2 + \sqrt{8} \textit{ or } d = -2 - \sqrt{8}$

The solution set is $\{-2 + \sqrt{8}, -2 - \sqrt{8}\}$.

Apply Quadratic Equations to Real-world Situations

3. A ball is thrown upward from the ground. Its height, h in feet, after t seconds is represented by the equation $h = 56t - 16t^2$. Determine how many seconds it will take before the ball returns to the ground.

The ball will hit the ground when its height is equal to 0 feet. Let $h = 0$*, and solve for* t*.*

$56t - 16t^2 = 0$

I need to use factoring to solve this equation because there is a t^2 and a t term.

$8t(7 - 2t) = 0$

$8t = 0 \textit{ or } 7 - 2t = 0$

$t = 0 \textit{ or } t = \frac{7}{2}$

The ball's height will be 0 feet when $t = 0$ *seconds or* $t = 3.5$ *seconds. The ball starts from a height of 0 feet and returns to the ground after* 3.5 *seconds.*

Lesson 7: Creating and Solving Quadratic Equations in One Variable

Application Problems Involving Quadratic Equations

I know that length · width $= 48$.

1. The width of a rectangle is 2 meters less than the length. The area is 48 square meters. Find the dimensions of the rectangle.

Let l represent the length of the rectangle.

Since the width is 2 meters less than the length, an expression for the width is $l - 2$.

$$l(l-2) = 48$$
$$l^2 - 2l - 48 = 0$$
$$(l-8)(l+6) = 0$$

To use factoring and the zero product property, one side of the equation must equal 0.

The solutions to this equation are 8 and -6.

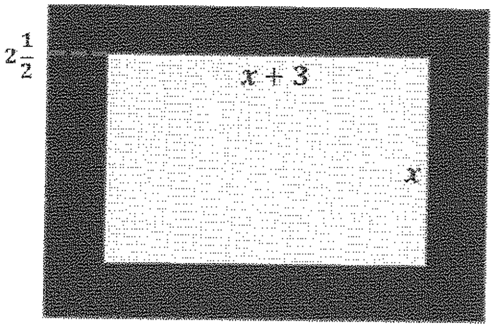
The length is 8 m and the width is 2 meters less than that number.

The dimensions of the rectangle are 8 m by 6 m.

I know that -6 cannot be a solution because length cannot be negative in this context.

2. A picture frame is $2\frac{1}{2}$ inches wide. The actual picture is 3 inches longer than its width. If the area of the picture is 180 square inches, what are the dimensions of the frame?

From the diagram, the picture's width is x inches, and its length is $x + 3$ inches.



$$x(x+3) = 180$$
$$x^2 + 3x = 180$$
$$x^2 + 3x - 180 = 0$$
$$(x+15)(x-12) = 0$$

The solutions to this equation are -15 and 12.

The width of the picture is 12 inches and the length is 15 inches ($12 + 3 = 15$).

The dimensions of the frame are as follows:

Width: $12 + 2.5 + 2.5 = 17$

Length: $15 + 2.5 + 2.5 = 20$

The width is 17 inches and the length is 20 inches.

I need to add 2.5 twice to both the length and the width of the picture to get the frame's dimensions.

3. Find two consecutive even integers whose product is 840.

Let w represent the first integer and $w + 2$ the next even integer.

From the example in class, I remember to add 2 to the first number to get the next consecutive even number.

$$w(w+2) = 840$$
$$w^2 + 2w - 840 = 0$$
$$(w+30)(w-28) = 0$$
$$w = -30 \text{ or } w = 28$$

To find the product, I need to multiply the integers.

The solutions to this equation are -30 and 28.

In this situation, a negative solution makes sense.

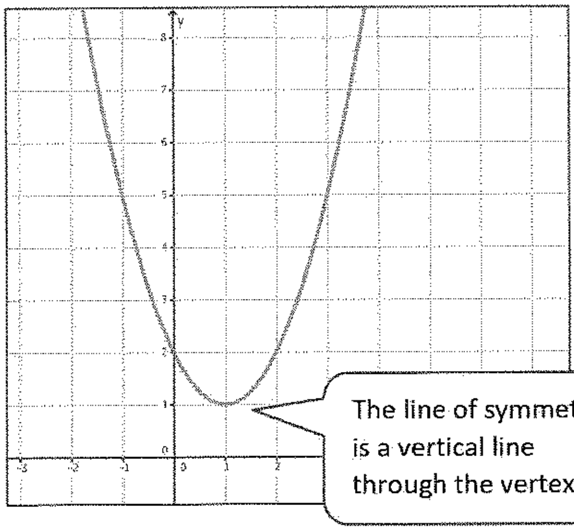
If the first integer is -30, then the next one is -28.

If the first integer is 28, then the next one is 30.

Lesson 8: Exploring the Symmetry in Graphs of Quadratic Functions

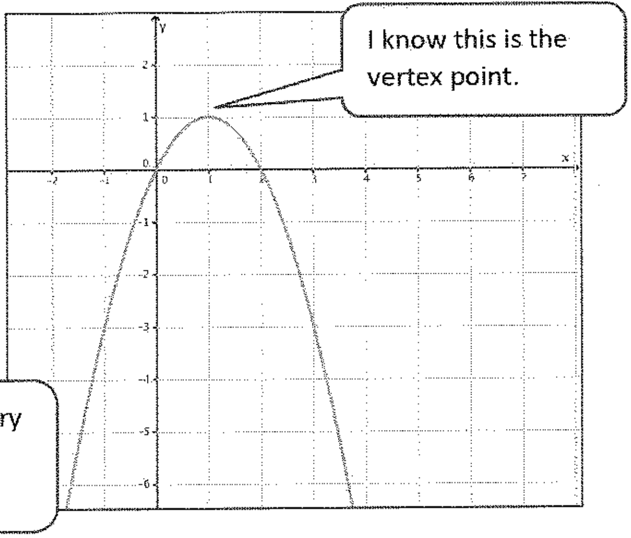
1. Consider the following key features discussed in this lesson for the four graphs of quadratic functions below: x-intercepts, y-intercepts, line of symmetry, vertex, and end behavior.

Graph A



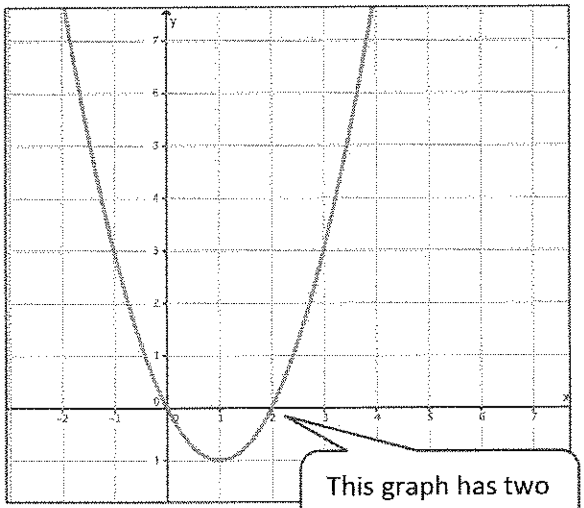
The line of symmetry is a vertical line through the vertex.

Graph B



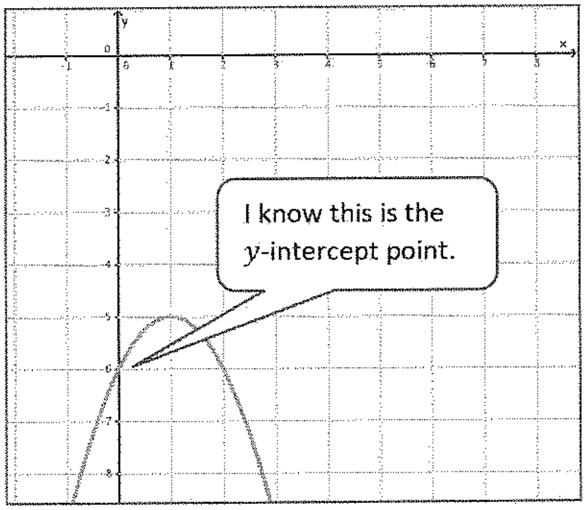
I know this is the vertex point.

Graph C



This graph has two x-intercepts.

Graph D



I know this is the y-intercept point.

a. Which key features of a quadratic function do graphs A and B have in common? Which features are not shared?

Same—line of symmetry and vertex

Different—y-intercept, x-intercepts, and end behavior

I see that graph A has no x-intercepts, and graph B has two x-intercepts.

b. Compare graphs B and C, and explain the differences and similarities between their key features.

Same—line of symmetry, y-intercept, x-intercepts

Different—vertex and end behavior

c. Compare graphs B and D, and explain the differences and similarities between their key features.

Same—line of symmetry and end behavior

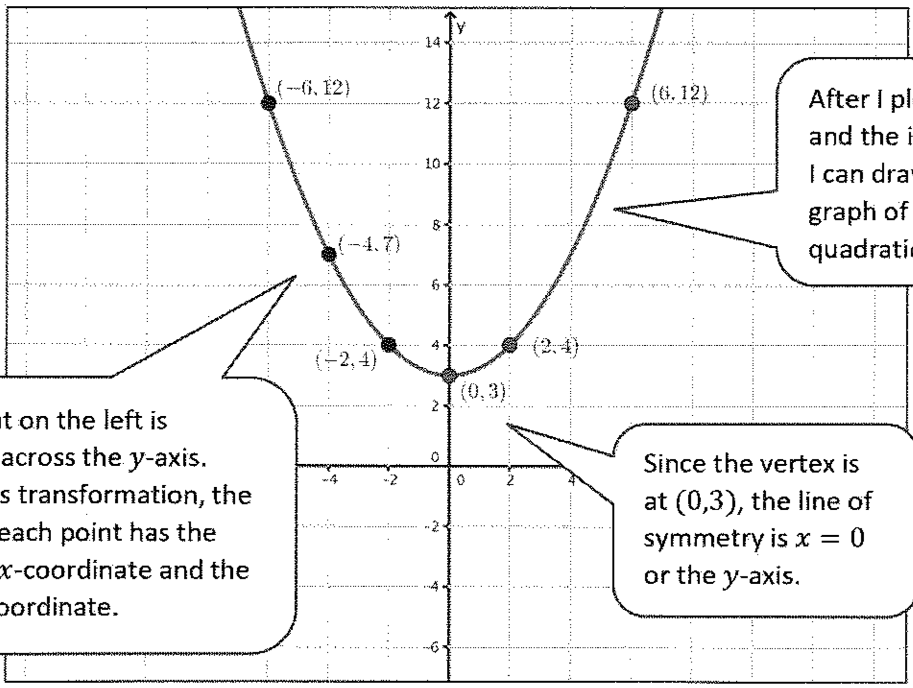
Different—y-intercept, x-intercepts, and vertex

d. What do all four of the graphs have in common?

They all have the same line of symmetry.

I see that all of the graphs have a line of symmetry of $x = 1$.

2. Use the symmetric properties of quadratic functions to sketch the graph of the function below, given these points and given that the vertex of the graph is the point $(0,3)$.



After I plot the vertex and the image points, I can draw in the graph of the quadratic function.

Each point on the left is reflected across the y-axis. Under this transformation, the image of each point has the opposite x-coordinate and the same y-coordinate.

Since the vertex is at $(0,3)$, the line of symmetry is $x = 0$ or the y-axis.

Lesson 9: Graphing Quadratic Functions from Factored Form, $f(x) = a(x - m)(x - n)$

Graphing Quadratic Functions in Factored Form

Graph the following on your own graph paper, and identify the key features of the graph.

1. $f(x) = -2(3x + 1)(3x + 1)$

The x-intercepts are solutions to $f(x) = 0$.

$-2(3x + 1)(3x + 1) = 0$ *when* $3x + 1 = 0$. *The solution is* $-\frac{1}{3}$.

The x-intercept is $-\frac{1}{3}$.

When a quadratic function has only one x-intercept, the vertex is the x-intercept point.

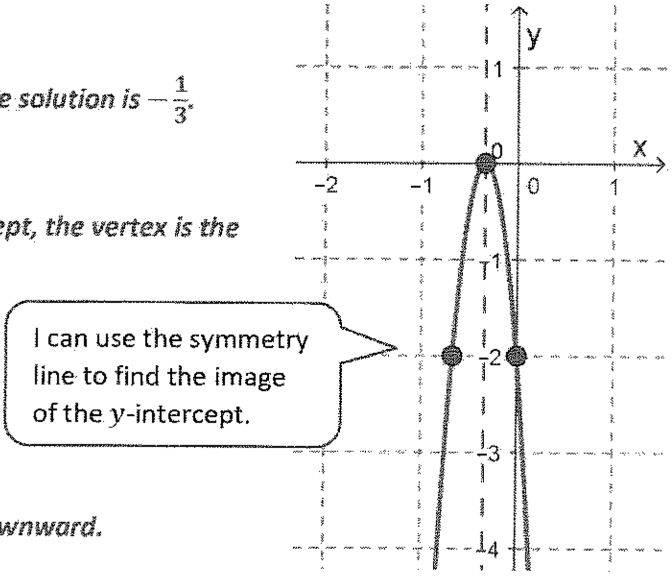
The vertex is $\left(-\frac{1}{3}, 0\right)$.

The y-intercept is $f(0)$.

$f(0) = -2(3 \cdot 0 + 1)(3 \cdot 0 + 1) = -2$

The y-intercept is -2.

The value of a is negative, so the graph opens downward.



I can use the symmetry line to find the image of the y-intercept.

2. $g(x) = x^2 - 9$

The x-intercepts are solutions to $g(x) = 0$.

$$x^2 - 9 = 0$$
$$(x + 3)(x - 3) = 0$$

The solutions are 3 and -3. The x-intercepts are 3 and -3.

The x-coordinate of the vertex is the average of the x-intercepts: $\frac{-3+3}{2} = 0$.

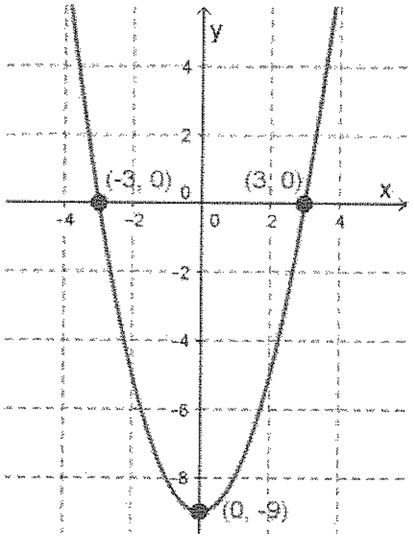
To find the y-coordinate of the vertex, evaluate g at $x = 0$.

$g(0) = 0^2 - 9 = -9$

The vertex is $(0, -9)$.

The y-intercept is $g(0) = -9$.

The value of a is positive, so the graph opens upward.



Applications of Graphs of Quadratic Functions in Factored Form

3. A diver jumps from a diving board into a pool. The height of the diver is given by the function $h(x) = -16x^2 + 8x + 24$, where the height h is measured in feet, and the time x is measured in seconds.

   a. Sketch the graph of the motion of the diver.

   Factored form: $h(x) = -8(2x^2 - x - 3) = -8(2x - 3)(x + 1)$

   I learned to factor when the a value is other than 1 in Lesson 3.

   Key features:

   The x-intercepts are 1.5 and -1.

   The y-intercept is 24.

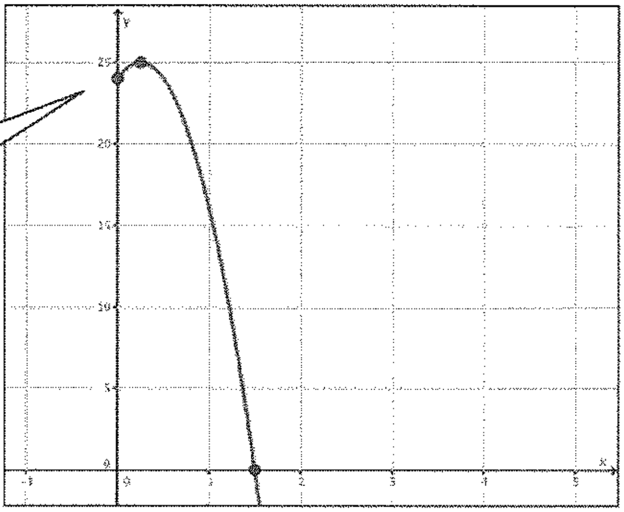
   The vertex x-coordinate is $\frac{1.5-1}{2} = 0.25$.

   $h(0.25) = 25$

   I can use a calculator to evaluate h when x is a rational number.

   Vertex: $(0.25, 25)$

   The graph opens downward because $a < 0$.



I don't need to plot the values where x is negative in this situation.

   b. When does the diver hit the water?

   The diver hits the water when h is 0. So, the diver hits the water at 1.5 seconds.

   c. When does the diver reach his maximum height?

   The diver reaches his maximum height at the vertex $(0.25, 25)$. The time is 0.25 seconds.

   d. What is the maximum height the diver reaches?

   The diver's maximum height is the y-coordinate of the vertex, 25 feet.

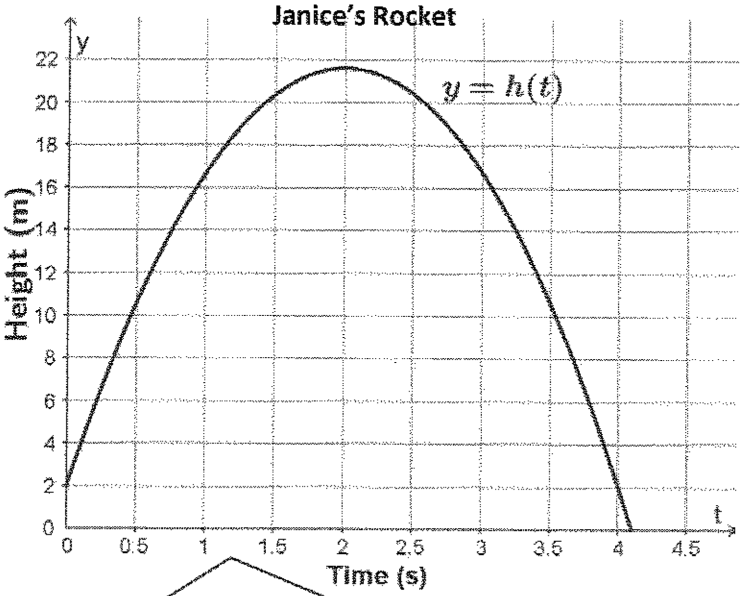
   e. What is the value of $h(0)$, and what does it mean for this problem?

   $h(0) = 24$. This means that the diver was at an initial height of 24 feet above the water. So, the height of the diving board was 24 feet high.

Lesson 10: Interpreting Quadratic Functions from Graphs and Tables

In algebra class, students launched model rockets to better understand quadratic functions. Janice's rocket launched at 19.6 meters per second (m/s) from a 2-meter tall platform. Kelley's rocket launched at 14.7 m/s from a 4-meter tall platform. The graph of $y = h(t)$ represents the height of Janice's rocket as a function of time since it was launched. A table that shows the height $g(t)$ of Kelley's rocket at various times t in seconds is also given. Use the graph and the table below to answer the questions.

Janice's Rocket



I can estimate the starting height, maximum height, and time when Janice's rocket will hit the ground from the graph.

Kelley's Rocket

t	$g(t)$
0	4.0
0.5	10.1
1	13.8
1.5	15.0
2	13.8
2.5	10.1
3	4.0
3.25	0.0

I can use the table to see the times and corresponding heights of Kelley's rocket.

a. According to the graph and table, when did each rocket reach its maximum height? Explain.

The graph shows that Janice's rocket reached its maximum height at 2 seconds. The line of symmetry for the graph is $t = 2$, which passes through the vertex. The t-coordinate of the points on the graph gives the times when the rocket is at each height, given by the y-coordinate of the points on the graph. In the table, $g(t)$ gives the height at time t. The largest height occurs when $t = 1.5$. I can see that the heights in the table are symmetric about this time, so I can conclude this is indeed the maximum height.

b. According to the graph and table, which rocket went higher into the air? Explain your answer.

The graph shows that the maximum height of Janice's rocket is approximately 21.6 meters. The table shows the maximum height for Kelley's rocket is 15 meters. Janice's rocket went higher.

c. How long was each rocket in the air?

The graph shows that Janice's rocket was in the air for approximately 4.1 seconds. The table shows that Kelley's rocket was in the air for 3.25 seconds.

From the table, the height is 0 at 3.25 seconds.

I observe that the graph shows that $t = 4.1$ when $y = 0$, which is the positive t-intercept or zero of the function.

d. Find $h(0)$ and $g(0)$, and explain what they mean in the problem.

The graph shows that $h(0) = 2$. The table shows that $g(0) = 4$. This means that Janice's rocket started at a height of 2 meters, and Kelley's rocket started at a height of 4 meters.