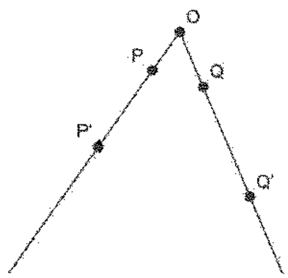
Homework Helpers

Grade 8
Module 3

G8-M3-Lesson 1: What Lies Behind "Same Shape"?

1. Let there be a dilation from center O. Then, $Dilation(P) = P'$, and $Dilation(Q) = Q'$. Examine the drawing below. What can you determine about the scale factor of the dilation?

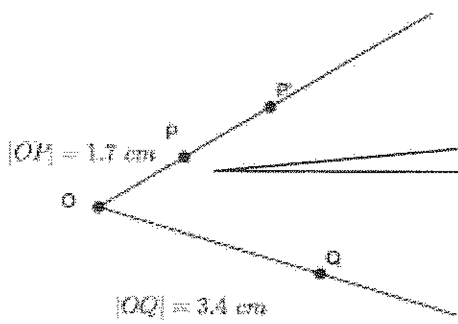


> I remember from the last module that the original points are labeled without primes, and the images are labeled with primes.

The dilation must have a scale factor larger than 1, $r > 1$, since the dilated points are farther from the center than the original points.

2. Let there be a dilation from center O with a scale factor $r = 2$. Then, $Dilation(P) = P'$, and $Dilation(Q) = Q'$. $|OP| = 1.7$ cm, and $|OQ| = 3.4$ cm, as shown. Use the drawing below to answer parts (a) and (b). The drawing is not to scale.

> I know that the bars around the segment represent length. So, $|OP'|$ is said, "The length of segment OP prime."



> We talked about the definition of dilation in class today. I should check the Lesson Summary box to review the definition.

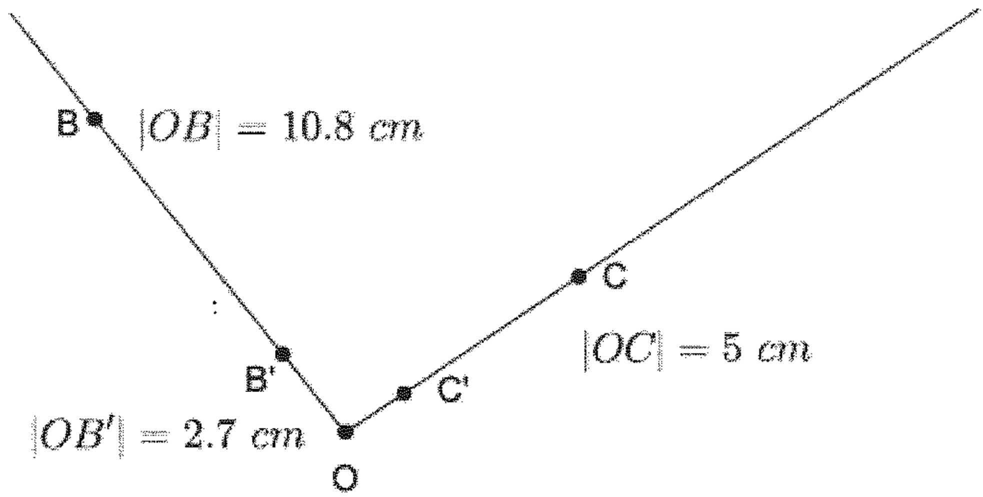
a. Use the definition of dilation to determine $|OP'|$.

$|OP'| = r|OP|$; therefore, $|OP'| = 2 \cdot (1.7) = 3.4$ and $|OP'| = 3.4$ cm.

b. Use the definition of dilation to determine $|OQ'|$.

$|OQ'| = r|OQ|$; therefore, $|OQ'| = 2 \cdot (3.4) = 6.8$ and $|OQ'| = 6.8$ cm.

3. Let there be a dilation from center O with a scale factor r. Then, $Dilation(B) = B'$, $Dilation(C) = C'$, and $|OB| = 10.8$ cm, $|OC| = 5$ cm, and $|OB'| = 2.7$ cm, as shown. Use the drawing below to answer parts (a)–(c).



a. Using the definition of dilation with $|OB|$ and $|OB'|$, determine the scale factor of the dilation.

$|OB'| = r|OB|$, which means $2.7 = r \cdot (10.8)$, then

$$\frac{2.7}{10.8} = r$$
$$\frac{1}{4} = r$$

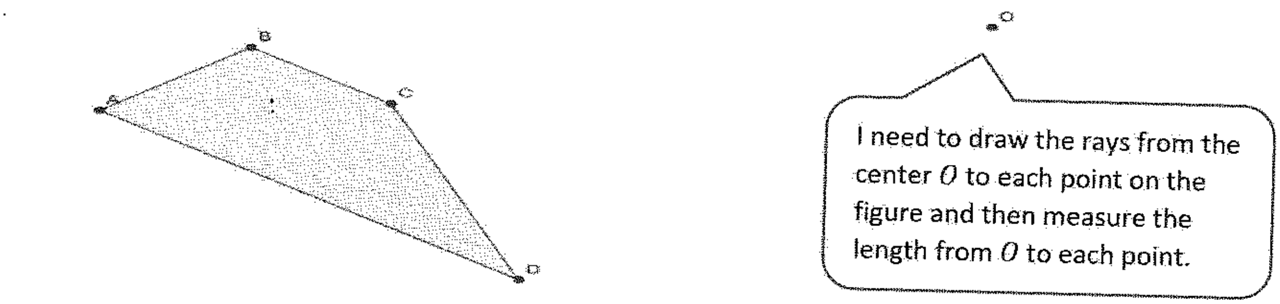
Since $|OB'| = r|OB|$, then by the multiplication property of equality, $\frac{|OB'|}{|OB|} = r$.

b. Use the definition of dilation to determine $|OC'|$.

Since the scale factor, r, is $\frac{1}{4}$, then $|OC'| = \frac{1}{4}|OC|$; therefore, $|OC'| = \frac{1}{4} \cdot 5 = 1.25$, and $|OC'| = 1.25$ cm.

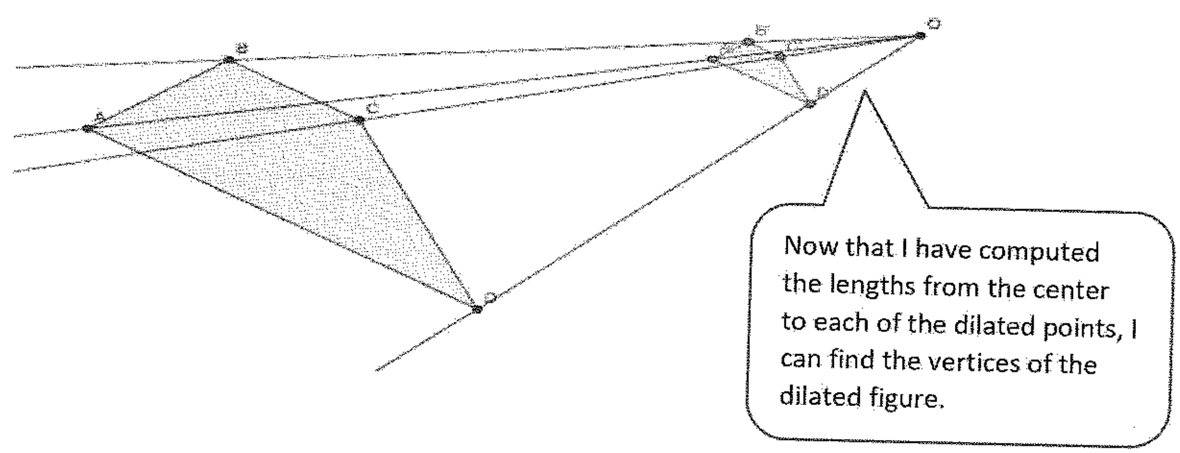
G8-M3-Lesson 2: Properties of Dilations

1. Given center O and quadrilateral $ABCD$, use a ruler to dilate the figure from center O by a scale factor of $r = \frac{1}{4}$. Label the dilated quadrilateral $A'B'C'D'$.

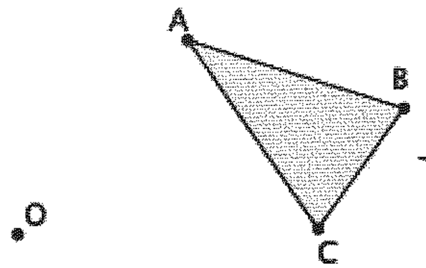


The figure in red below shows the dilated image of $ABCD$. All measurements are in centimeters.

$\lvert OA'\rvert = r\lvert OA\rvert$	$\lvert OB'\rvert = r\lvert OB\rvert$	$\lvert OC'\rvert = r\lvert OC\rvert$	$\lvert OD'\rvert = r\lvert OD\rvert$
$\lvert OA'\rvert = \frac{1}{4}(11.2)$	$\lvert OB'\rvert = \frac{1}{4}(9.2)$	$\lvert OC'\rvert = \frac{1}{4}(7.6)$	$\lvert OD'\rvert = \frac{1}{4}(6.8)$
$\lvert OA'\rvert = 2.8$	$\lvert OB'\rvert = 2.3$	$\lvert OC'\rvert = 1.9$	$\lvert OD'\rvert = 1.7$

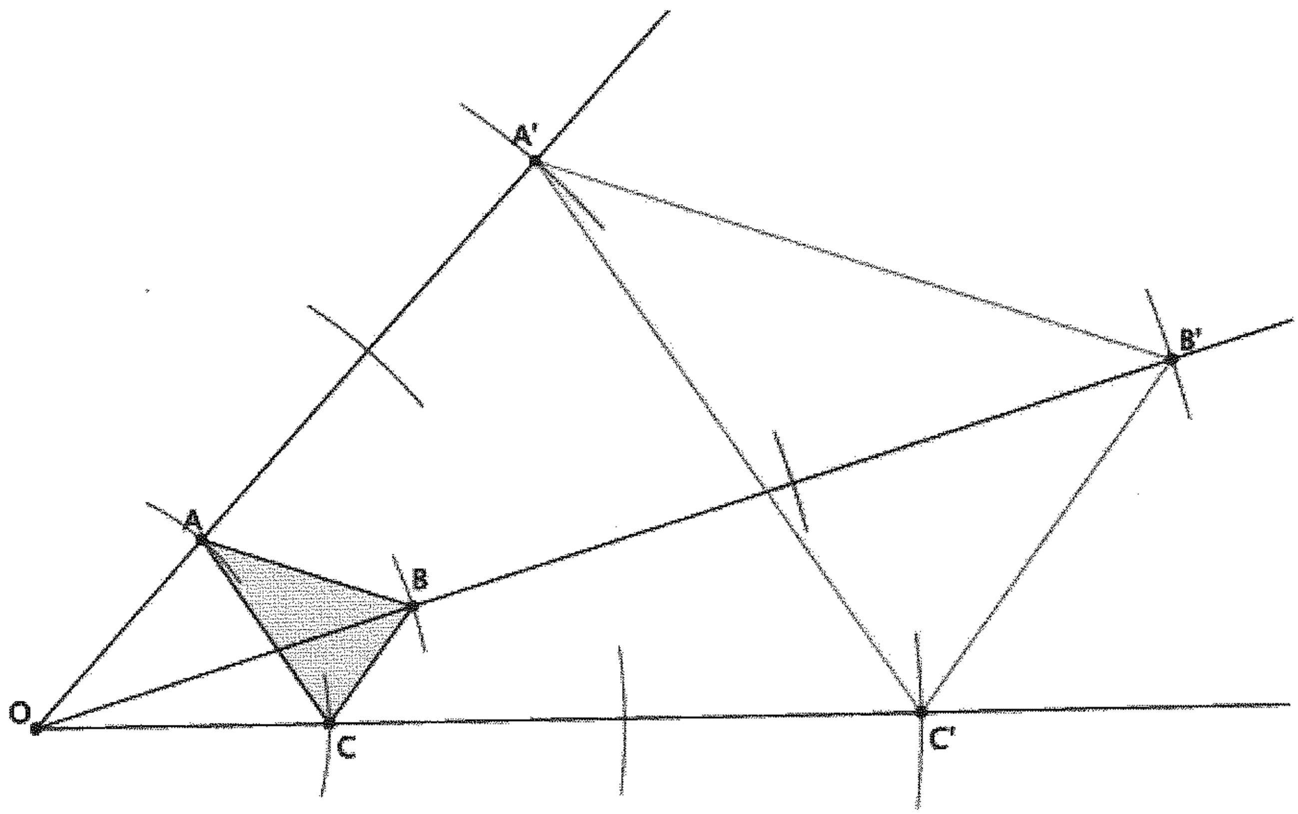


2. Use a compass to dilate the figure ABC from center O, with scale factor $r = 3$.



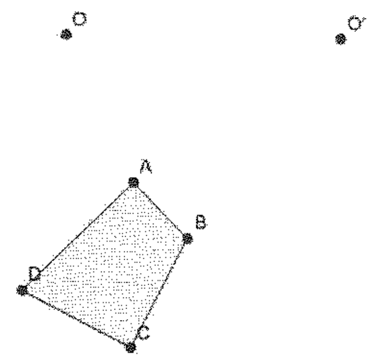
I need to draw the rays like before, but this time I can use a compass to measure the distance from the center O to a point and again use the compass to find the length 3 times from the center.

The figure in red below shows the dilated image of ABC.

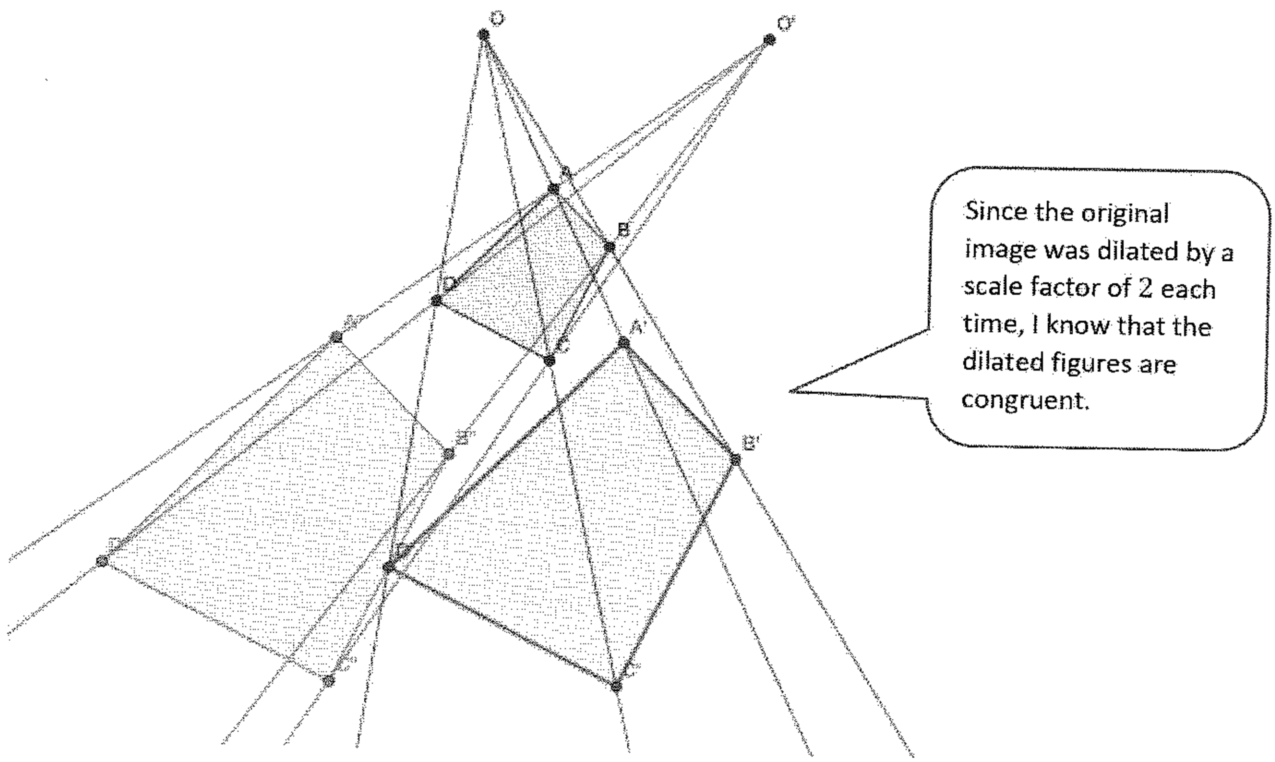


3. Use a compass to dilate the figure $ABCD$ from center O, with scale factor $r = 2$.

   a. Dilate the same figure, $ABCD$, from a new center, O', with scale factor $r = 2$. Use double primes ($A''B''C''D''$) to distinguish this image from the original.



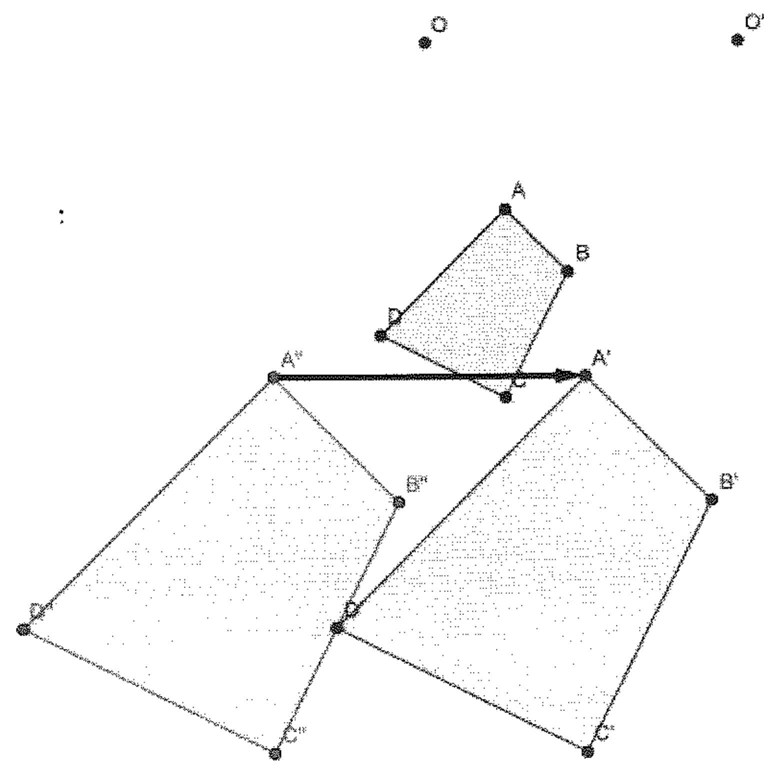
The figure in blue, $A'B'C'D'$, shows the dilation of $ABCD$ from center O, with scale factor $r = 2$. The figure in red, $A''B''C''D''$, shows the dilation of $ABCD$ from center O' with scale factor $r = 2$.



b. What rigid motion, or sequence of rigid motions, would map $A''B''C''D''$ to $A'B'C'D'$?

A translation along vector $\overrightarrow{A''A'}$ (or any vector that connects a point of $A''B''C''D''$ and its corresponding point of $A'B'C'D'$) would map the figure $A''B''C''D''$ to $A'B'C'D'$.

The image below (with rays removed for clarity) shows the vector $\overrightarrow{A''A'}$.



4. A line segment AB undergoes a dilation. Based on today's lesson, what is the image of the segment?

   The segment dilates as a segment.

5. $\angle AOC$ measures $24°$. After a dilation, what is the measure of $\angle A'OC'$? How do you know?

   The measure of $\angle A'OC'$ is $24°$. Dilations preserve angle measure, so $\angle A'OC'$ remains the same measure as $\angle AOC$.

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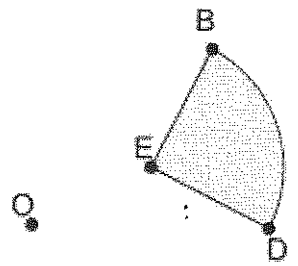
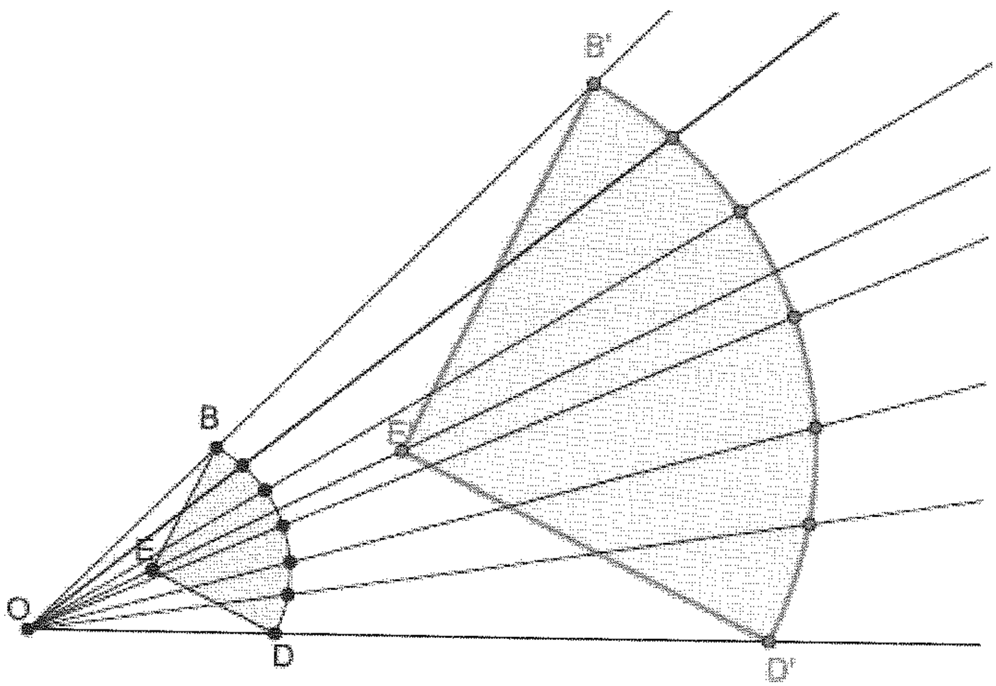
G8-M3-Lesson 3: Examples of Dilations

1. Dilate the figure from center O by a scale factor of $r = 3$. Make sure to use enough points to make a good image of the original figure.



If I only dilate the points B, E, and D, then the dilated figure will look like a triangle. I will need to dilate more points along the curve BD.

The dilated image is shown in red below. Several points are needed along the curved portion of the diagram to produce an image similar to the original.



2. A triangle ABC was dilated from center O by a scale factor of $r = 8$. What scale factor would shrink the dilated figure back to the original size?

   A scale factor of $r = \frac{1}{8}$ *would bring the dilated figure back to its original size.*

   > To dilate back to its original size, I need to use the reciprocal of the original scale factor because $r \cdot \frac{1}{r} = 1$.

3. A figure has been dilated from center O by a scale factor of $r = \frac{2}{3}$. What scale factor would shrink the dilated figure back to the original size?

   A scale factor of $r = \frac{3}{2}$ *would bring the dilated figure back to its original size.*

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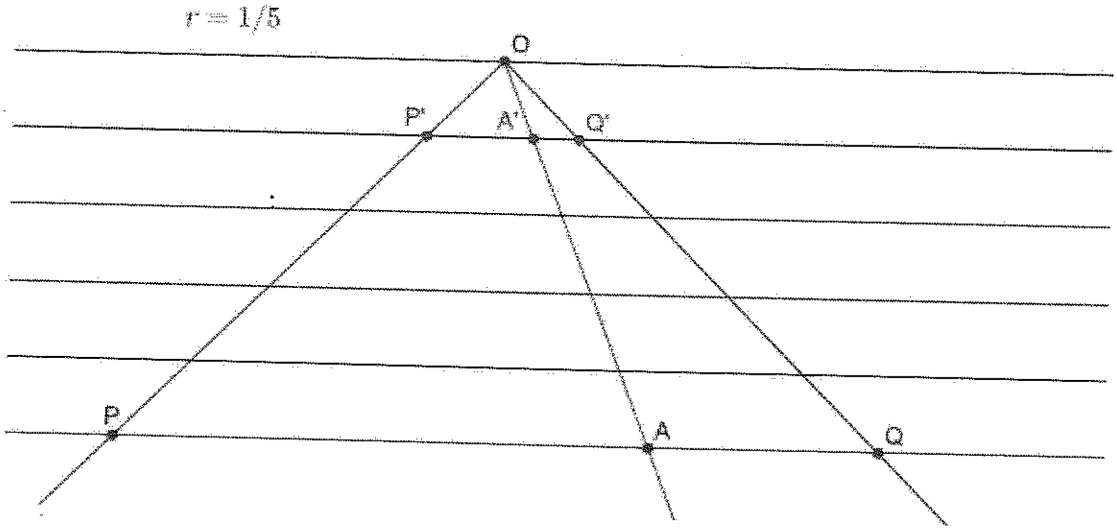
G8-M3-Lesson 4: Fundamental Theorem of Similarity (FTS)

1. In the diagram below, points P, Q, and A have been dilated from center O by a scale factor of $r = \frac{1}{5}$ on a piece of lined paper.



a. What is the relationship between segments PQ and $P'Q'$? How do you know?

$\overline{PQ}$ and $\overline{P'Q'}$ are parallel because they follow the lines on the lined paper.

b. What is the relationship between segments PA and $P'A'$? How do you know?

$\overline{PA}$ and $\overline{P'A'}$ are also parallel because they follow the lines on the lined paper.

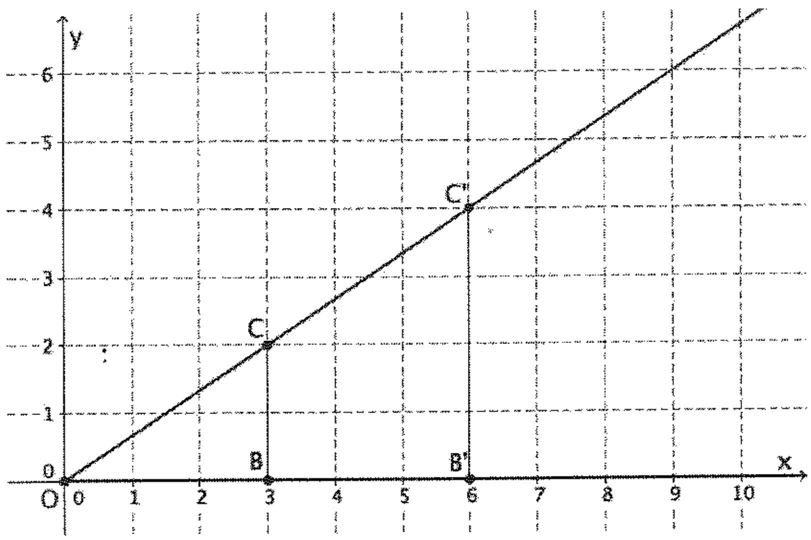
c. Identify two angles whose measures are equal. How do you know?

$\angle OAP$ and $\angle OA'P'$ are equal in measure because they are corresponding angles created by parallel lines AP and $A'P'$.

d. What is the relationship between the lengths of segments AQ and $A'Q'$? How do you know?

The length of segment $A'Q'$ will be $\frac{1}{5}$ the length of segment AQ. The FTS states that the length of the dilated segment is equal to the scale factor multiplied by the original segment length, or $|A'Q'| = r|AQ|$.

2. Reynaldo sketched the following diagram on graph paper. He dilated points B and C from center O.



a. What is the scale factor r? Show your work.

$$|OB'| = r|OB|$$
$$6 = r(3)$$
$$\frac{6}{3} = r$$
$$2 = r$$

b. Verify the scale factor with a different set of segments.

$$|B'C'| = r|BC|$$
$$4 = r(2)$$
$$\frac{4}{2} = r$$
$$2 = r$$

c. Which segments are parallel? How do you know?

Segments BC and $B'C'$ are parallel. They are on the lines of the grid paper, which I know are parallel.

d. Which angles are equal in measure? How do you know?

$|\angle OB'C'| = |\angle OBC|$, and $|\angle OC'B'| = |\angle OCB|$ because they are corresponding angles of parallel lines cut by a transversal.

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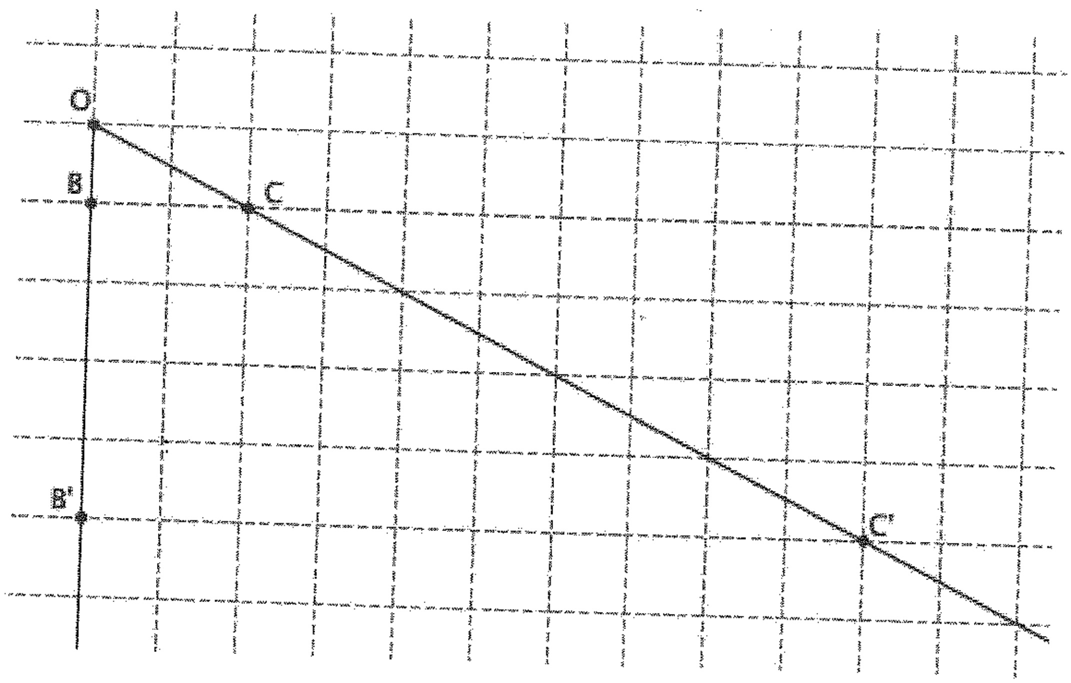
3. Points B and C were dilated from center O.



a. What is the scale factor r? Show your work.

$$|OB'| = r|OB|$$
$$5 = r(1)$$
$$5 = r$$

b. If $|OC| \approx 2.2$, what is $|OC'|$?

$$|OC'| = r|OC|$$
$$|OC'| \approx 5(2.2)$$
$$|OC'| \approx 11$$

c. How does the perimeter of $\triangle\ OBC$ compare to the perimeter of $\triangle\ OB'C'$?

Perimeter $\triangle\ OBC \approx 1 + 2 + 2.2$

Perimeter $\triangle\ OBC \approx 5.2$

Perimeter $\triangle\ OB'C' \approx 5 + 10 + 11$

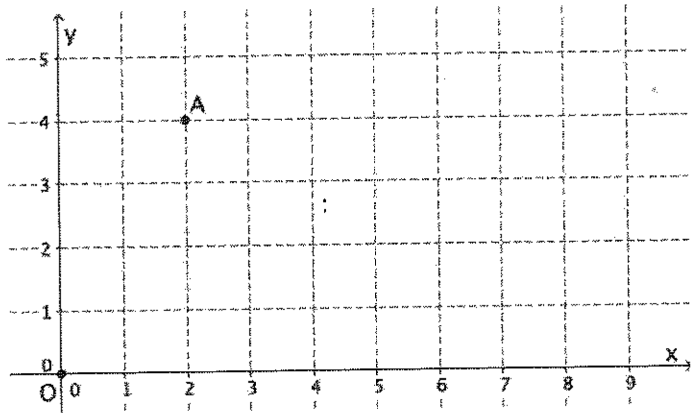
Perimeter $\triangle\ OB'C' \approx 26$

d. Was the perimeter of $\triangle\ OB'C'$ equal to the perimeter of $\triangle\ OBC$ multiplied by scale factor r? Explain.

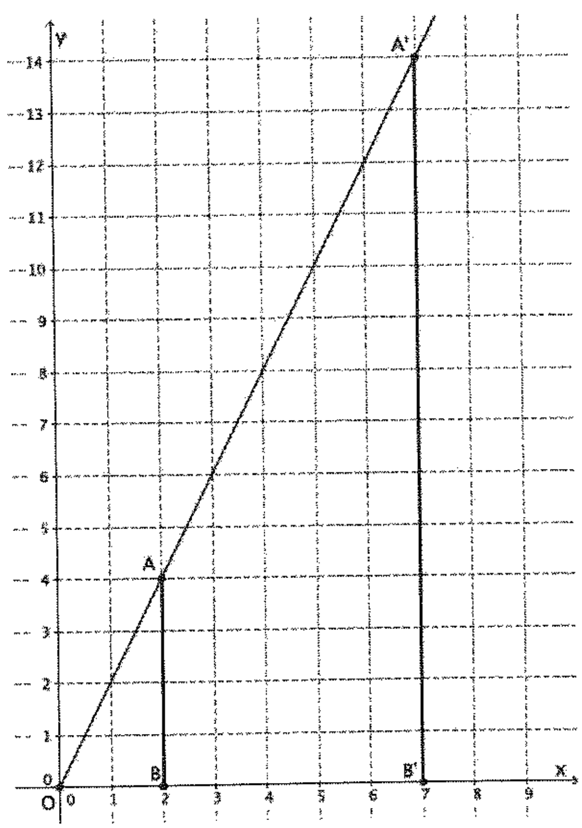
Yes. The perimeter of $\triangle\ OB'C'$ was five times the perimeter of $\triangle\ OBC$, which makes sense because the dilation increased the length of each segment by a scale factor of 5. That means that each side of $\triangle\ OB'C'$ was five times as long as each side of $\triangle\ OBC$.

G8-M3-Lesson 5: First Consequences of FTS

1. Dilate point A, located at $(2, 4)$ from center O, by a scale factor of $r = \frac{7}{2}$. What is the precise location of point A'?



When we did this in class, I know we started by marking a point B on the x-axis on the same vertical line as point A. Then we used what we learned in the last lesson. I should review my classwork.



$|OB'| = r|OB|$

$|OB'| = \frac{7}{2}(2)$

$|OB'| = 7$

Now I know which vertical line A' will fall on; it is the same as where B' is.

$|A'B'| = r|AB|$

$|A'B'| = \frac{7}{2}(4)$

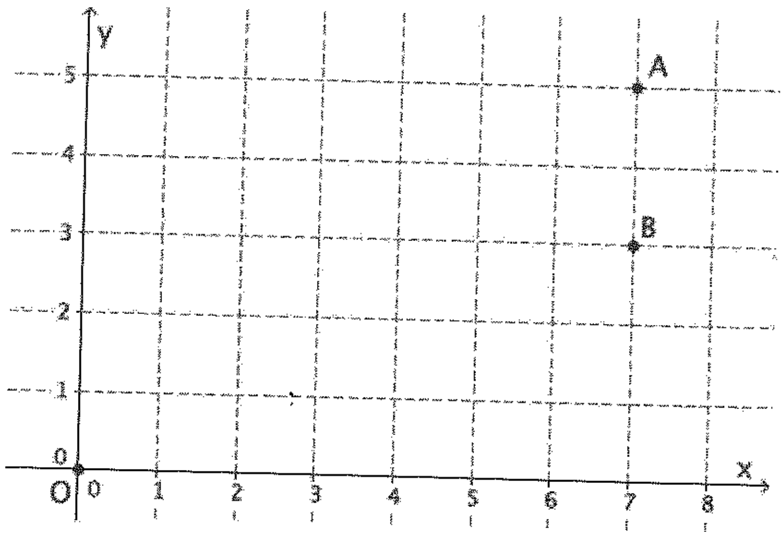
$|A'B'| = \frac{28}{2}$

$|A'B'| = 14$

I know that segment $A'B'$ is a dilation of segment AB by the same scale factor $r = \frac{7}{2}$. Since $\overline{AB}$ is contained within a vertical line, I can easily determine its length is 4 units.

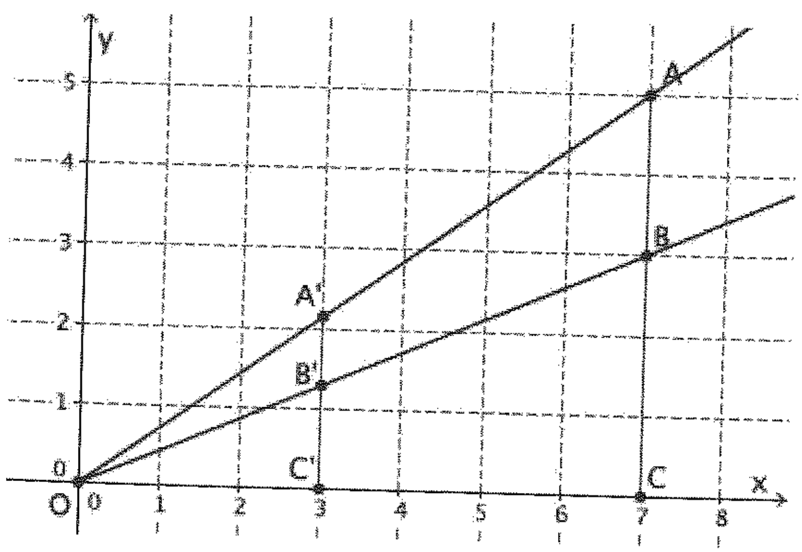
Therefore, A' is located at $(7, 14)$.

2. Dilate point A, located at $(7, 5)$ from center O, by a scale factor of $r = \frac{3}{7}$. Then, dilate point B, located at $(7, 3)$ from center O, by a scale factor of $r = \frac{3}{7}$. What are the coordinates of A' and B'? Explain.



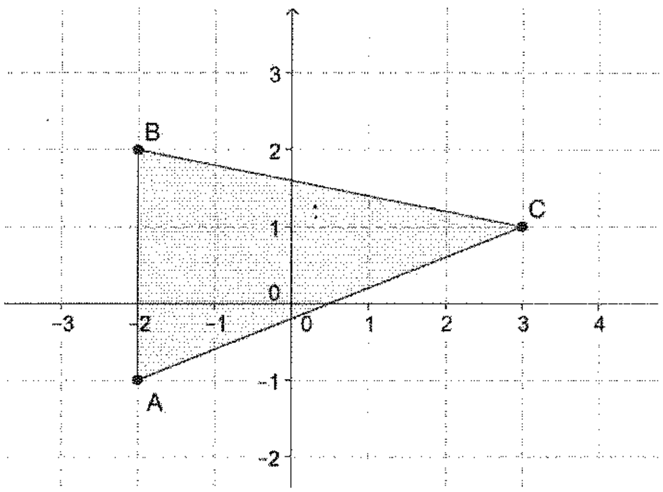
> This is just like the last problem, but now I have to find coordinates for two points. I'll need to mark a point C on the x-axis on the same vertical line as points A and B.

The y-coordinate of point A' is the same as the length of $\overline{A'C'}$. Since $|A'C'| = r|AC|$, then $|A'C'| = \frac{3}{7} \cdot 5 = \frac{15}{7}$. The location of point A' is $\left(3, \frac{15}{7}\right)$, or approximately $(3, 2.1)$. The y-coordinate of point B' is the same as the length of $\overline{B'C'}$. Since $|B'C'| = r|BC|$, then $|B'C'| = \frac{3}{7} \cdot 3 = \frac{9}{7}$. The location of point B' is $\left(3, \frac{9}{7}\right)$, or approximately $(3, 1.3)$.



G8-M3-Lesson 6: Dilations on the Coordinate Plane

1. Triangle ABC is shown on the coordinate plane below. The triangle is dilated from the origin by scale factor $r = 2$. Identify the coordinates of the dilated triangle $A'B'C'$.



The work we did in class led us to the conclusion that when given a point $A(x, y)$, we can find the coordinates of A' using the scale factor: $A'(rx, ry)$. This only works for dilations from the origin.

$A(-2,-1) \rightarrow A'(2 \cdot (-2), 2 \cdot (-1)) = A'(-4,-2)$
$B(-2,2) \rightarrow B'(2 \cdot (-2), 2 \cdot 2) = B'(-4,4)$
$C(3,1) \rightarrow C'(2 \cdot 3, 2 \cdot 1) = C'(6,2)$

The coordinates of the dilated triangle will be $A'(-4,-2)$, $B'(-4,4)$, $C'(6,2)$.

2. The figure $ABCD$ has coordinates $A(3,1)$, $B(12,9)$, $C(-9,3)$, and $D(-12,-3)$. The figure is dilated from the origin by a scale factor $r = \frac{2}{3}$. Identify the coordinates of the dilated figure $A'B'C'D'$.

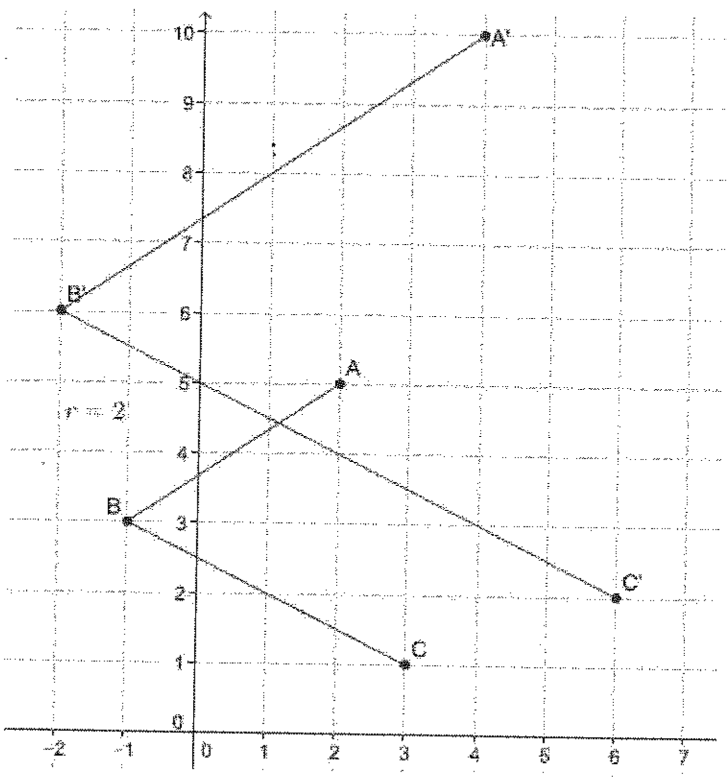
$$A(3,1) \rightarrow A'\left(\frac{2}{3} \cdot 3, \frac{2}{3} \cdot 1\right) = A'\left(2, \frac{2}{3}\right)$$
$$B(12,9) \rightarrow B'\left(\frac{2}{3} \cdot 12, \frac{2}{3} \cdot 9\right) = B'(8,6)$$
$$C(-9,3) \rightarrow C'\left(\frac{2}{3} \cdot (-9), \frac{2}{3} \cdot 3\right) = C'(-6,2)$$
$$D(-12,-3) \rightarrow D'\left(\frac{2}{3} \cdot (-12), \frac{2}{3} \cdot (-3)\right) = D'(-8,-2)$$

The coordinates of the dilated figure are $A'\left(2, \frac{2}{3}\right)$, $B'(8,6)$, $C'(-6, 2)$, and $D'(-8,-2)$.

G8-M3-Lesson 7: Informal Proofs of Properties of Dilation

1. A dilation from center O by scale factor r of an angle maps to what? Verify your claim on the coordinate plane.

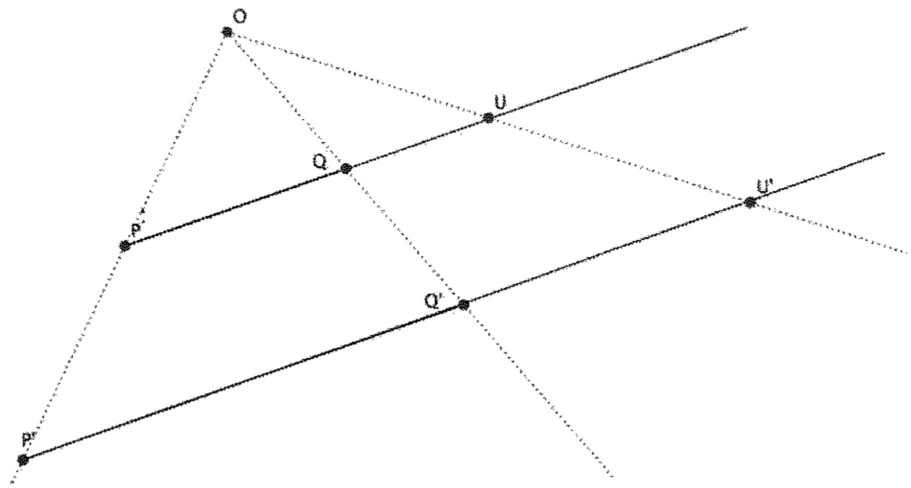
The dilation of an angle maps to an angle.



To verify, I can choose any three points to create the angle and any scale factor. I can use what I learned in the last lesson to find the coordinates of the dilated points.

2. Prove the theorem: A dilation maps rays to rays.

Let there be a dilation from center O with scale factor r so that $P' = Dilation(P)$ and $Q' = Dilation(Q)$. Show that ray PQ maps to ray $P'Q'$ (i.e., that dilations map rays to rays). Using the diagram, answer the questions that follow, and then informally prove the theorem. (Hint: This proof is a lot like the proof for segments. This time, let U be a point on line PQ that is not between points P and Q.)



a. U is a point on $\overrightarrow{PQ}$. By definition of dilation what is the name of $Dilation(U)$?

By the definition of dilation, we know that $U' = Dilation(U)$.

b. By the definition of dilation we know that $\frac{|OP'|}{|OP|} = r$. What other two ratios are also equal to r?

By the definition of dilation, we know that $\frac{|OQ'|}{|OQ|} = \frac{|OU'|}{|OU|} = r$.

c. By FTS, what do we know about line PQ and line $P'Q'$?

By FTS, we know that line PQ and line $P'Q'$ are parallel.

d. What does FTS tell us about line QU and line $Q'U'$?

By FTS, we know that line QU and line $Q'U'$ are parallel.

e. What conclusion can be drawn about the line that contains $\overrightarrow{PQ}$ and the line $Q'U'$?

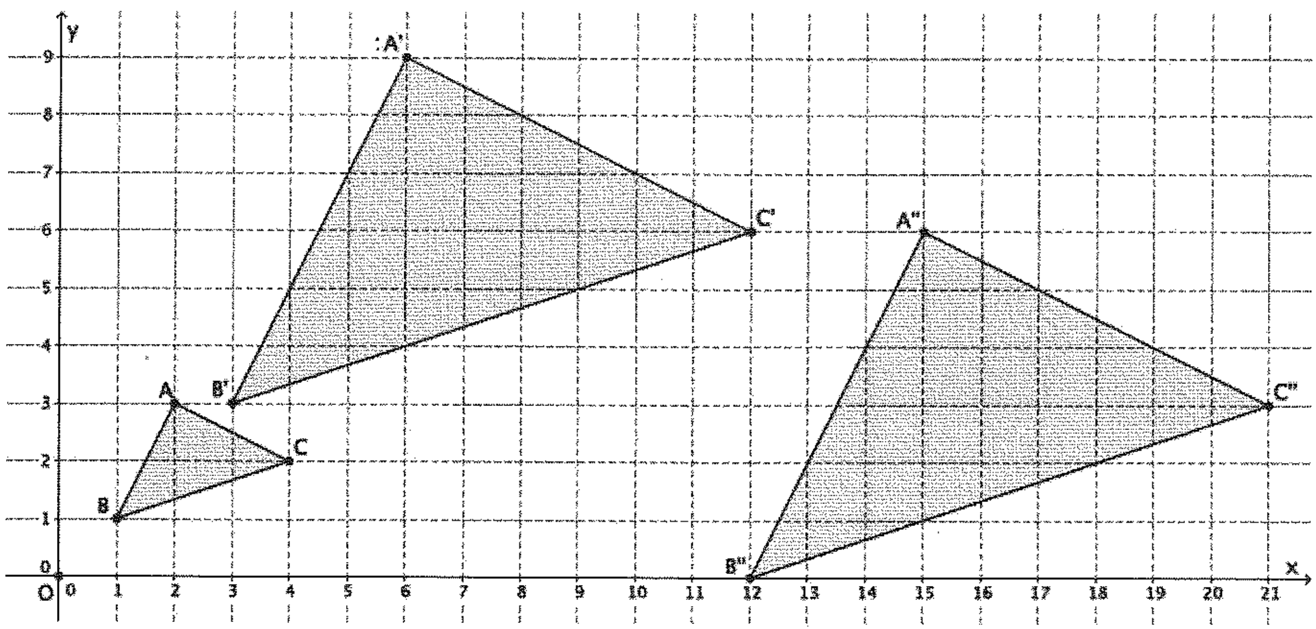
The line that contains $\overrightarrow{PQ}$ is parallel to $\overleftrightarrow{Q'U'}$ because $\overleftrightarrow{Q'U'}$ is contained in the line $P'Q'$.

f. Informally prove that $\overrightarrow{P'Q'}$ is a dilation of $\overrightarrow{PQ}$.

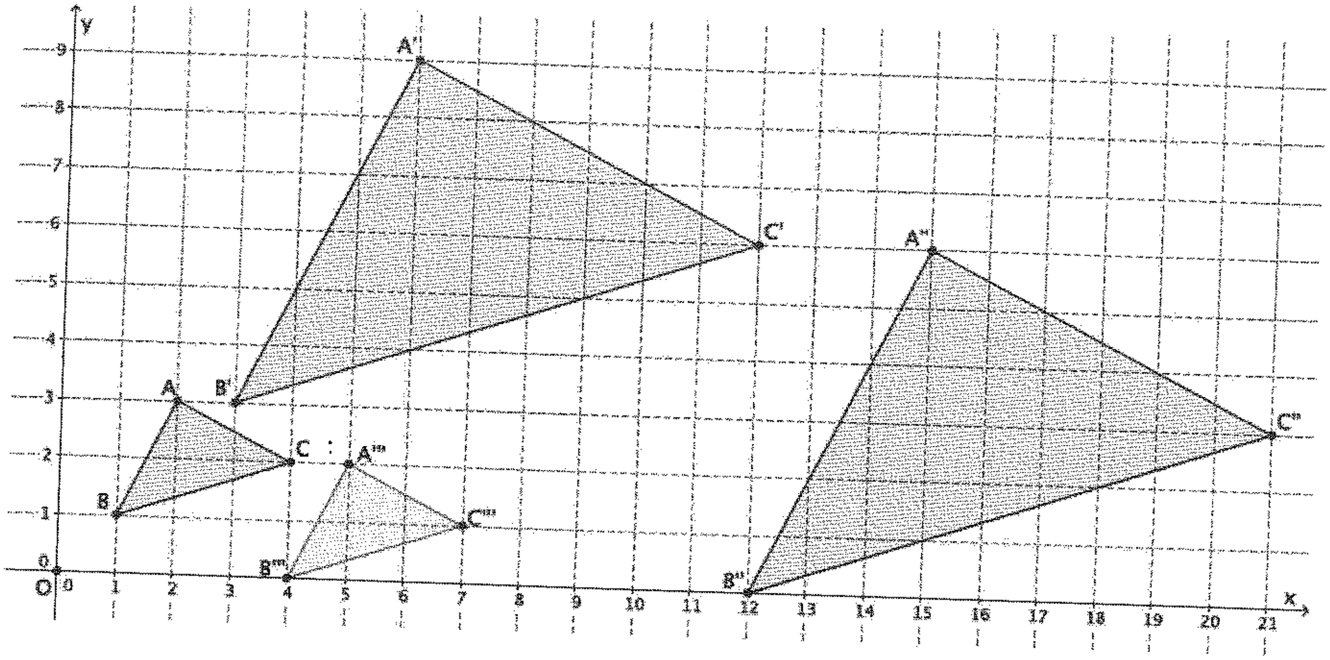
Using the information from parts (a)–(e), we know that U is a point on $\overrightarrow{PQ}$. We also know that the line that contains $\overrightarrow{PQ}$ is parallel to line $Q'U'$. But we already know that $\overrightarrow{PQ}$ is parallel to $\overleftrightarrow{P'Q'}$. Since there can only be one line that passes through Q' that is parallel to $\overrightarrow{PQ}$, then the line that contains $\overrightarrow{P'Q'}$ and line $Q'U'$ must coincide. That places the dilation of point U, U', on $\overrightarrow{P'Q'}$, which proves that dilations map rays to rays.

G8-M3-Lesson 8: Similarity

1. In the picture below, we have a triangle ABC that has been dilated from center O by scale factor $r = 3$. It is noted by $A'B'C'$. We also have a triangle $A''B''C''$, which is congruent to triangle $A'B'C'$ (i.e., $\triangle A'B'C' \cong \triangle A''B''C''$). Describe the sequence of a dilation, followed by a congruence (of one or more rigid motions), that would map triangle $A''B''C''$ onto triangle ABC.

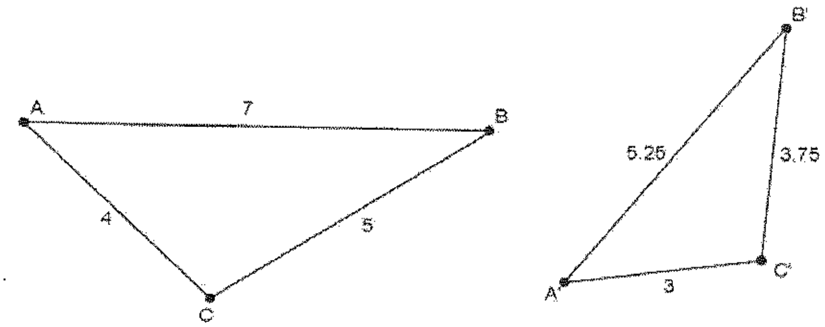


First, we must dilate triangle $A''B''C''$ from center O by scale factor $r = \frac{1}{3}$ to shrink it to the size of triangle ABC. I will note this triangle as $A'''B'''C'''$. Once I have the triangle to the right size, I can translate the dilated triangle, $A'''B'''C'''$, up one unit and to the left three units.



First, we must dilate triangle $A''B''C''$ from center O by scale factor $r = \frac{1}{3}$ to shrink it to the size of triangle ABC. Next, we must translate the dilated triangle, noted by $A'''B'''C'''$, one unit up and three units to the left. This sequence of the dilation followed by the translation would map triangle $A''B''C''$ onto triangle ABC.

2. Triangle ABC is similar to triangle $A'B'C'$ (i.e., $\triangle ABC \sim \triangle A'B'C'$). Prove the similarity by describing a sequence that would map triangle $A'B'C'$ onto triangle ABC.



> I can check the ratios of the corresponding sides to see if they are the same proportion and equal to the same scale factor.

The scale factor that would magnify triangle $A'B'C'$ to the size of triangle ABC is $r = \frac{4}{3}$.

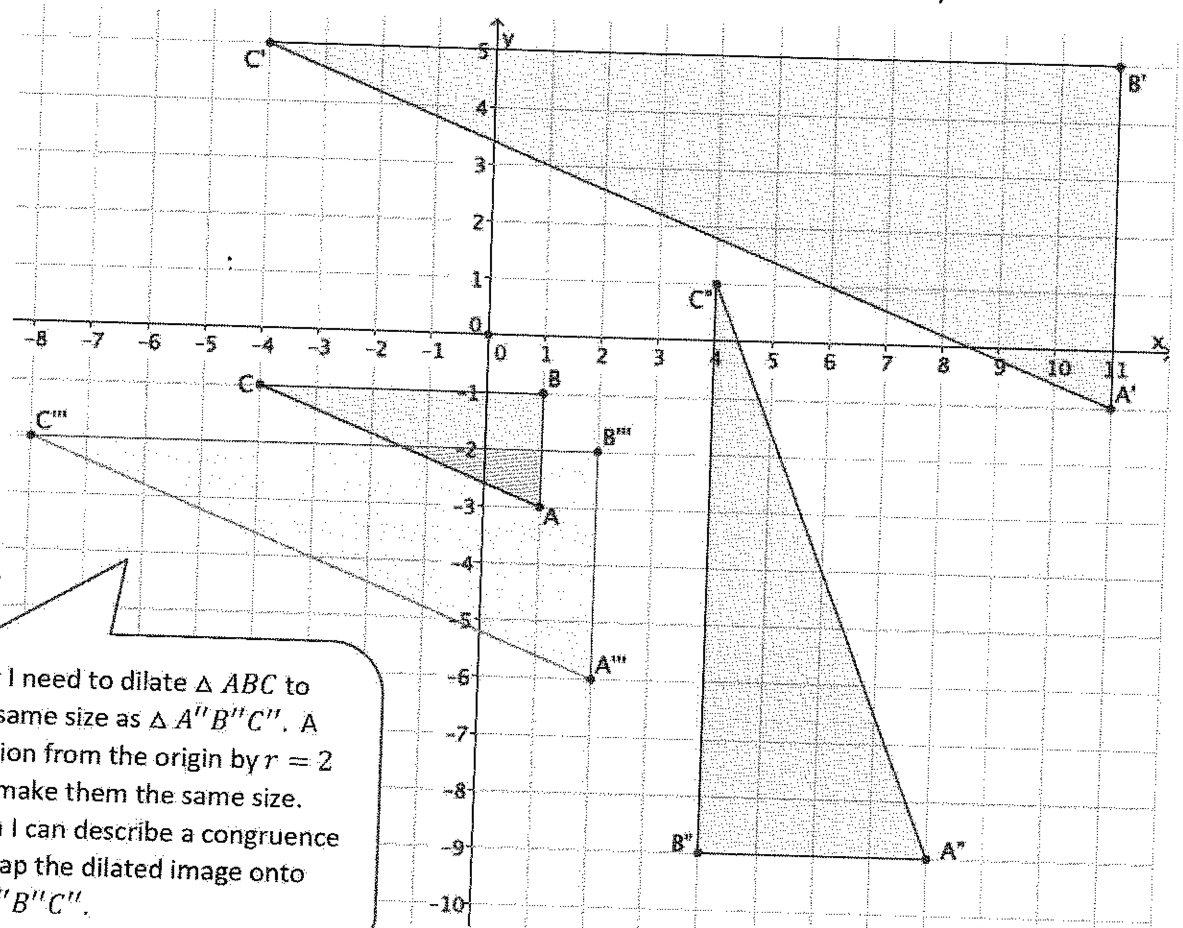
> Once the triangle $A'B'C'$ is the same size as triangle ABC, I can describe a congruence to map triangle $A'B'C'$ onto triangle ABC.

Sample description:

The sequence that would prove the similarity of the triangles is a dilation from a center by a scale factor of $r = \frac{4}{3}$, followed by a translation along vector $\overrightarrow{A'A}$, and finally, a rotation about point A.

G8-M3-Lesson 9: Basic Properties of Similarity

1. In the diagram below, $\triangle ABC \sim \triangle A'B'C'$ and $\triangle A'B'C' \sim \triangle A''B''C''$. Is $\triangle ABC \sim \triangle A''B''C''$? If so, describe the dilation followed by the congruence that demonstrates the similarity.



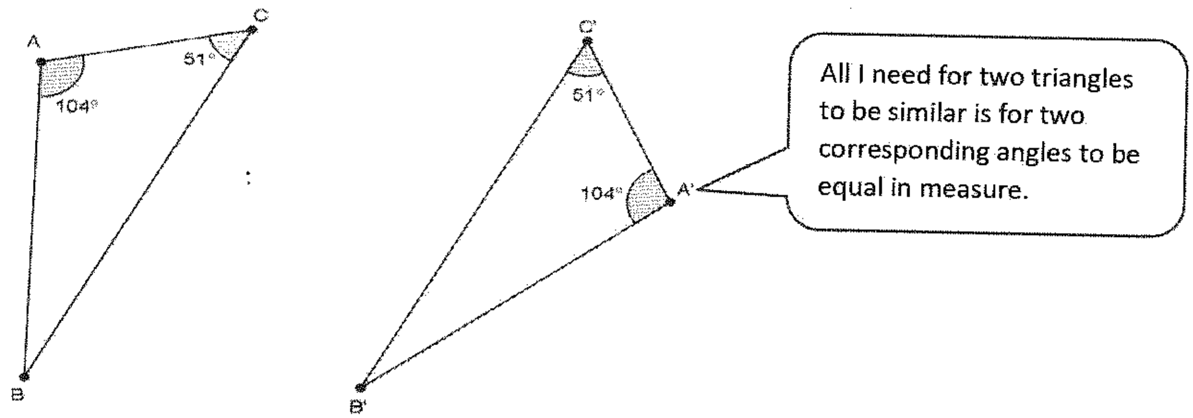
First I need to dilate $\triangle ABC$ to the same size as $\triangle A''B''C''$. A dilation from the origin by $r = 2$ will make them the same size. Then I can describe a congruence to map the dilated image onto $\triangle A''B''C''$.

Yes, $\triangle ABC \sim \triangle A''B''C''$ because similarity is transitive. Since $r|AB| = |A''B''|$, $|AB| = 2$, and $|A''B''| = 4$, then $r \cdot 2 = 4$. Therefore, $r = 2$. Then, a dilation from the origin by scale factor $r = 2$ makes $\triangle ABC$ the same size as $\triangle A''B''C''$. Translate the dilated image of $\triangle ABC$, $\triangle A'''B'''C'''$, 12 units to the right and 3 units up to map C''' to C''. Next, rotate the dilated image about point C'', 90 degrees clockwise. Finally, reflect the rotated image across line $C''B''$. The sequence of the dilation and the congruence map $\triangle ABC$ onto $\triangle A''B''C''$, demonstrating the similarity.

I remember that when I have to translate an image, it is best to do it so that corresponding points, like C and C'', coincide.

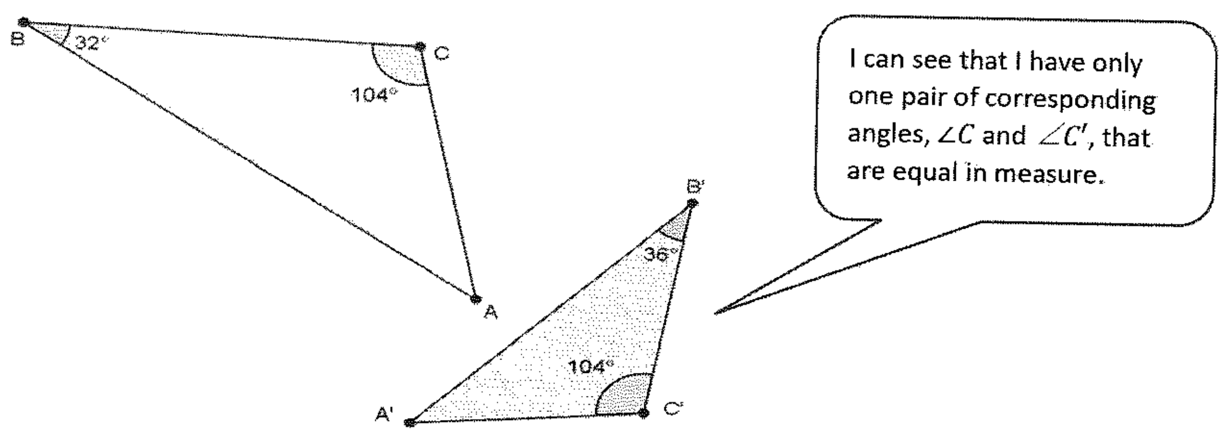
G8-M3-Lesson 10: Informal Proof of AA Criterion for Similarity

1. Are the triangles shown below similar? Present an informal argument as to why they are or are not similar.



Yes, $\triangle ABC \sim \triangle A'B'C'$. They are similar because they have two pairs of corresponding angles that are equal in measure, namely, $|\angle A| = |\angle A'| = 104°$, and $|\angle C| = |\angle C'| = 51°$.

2. Are the triangles shown below similar? Present an informal argument as to why they are or are not similar.



No, $\triangle ABC$ is not similar to $\triangle A'B'C'$. By the given information, $|\angle B| \neq |\angle B'|$, and $|\angle A| \neq |\angle A'|$.

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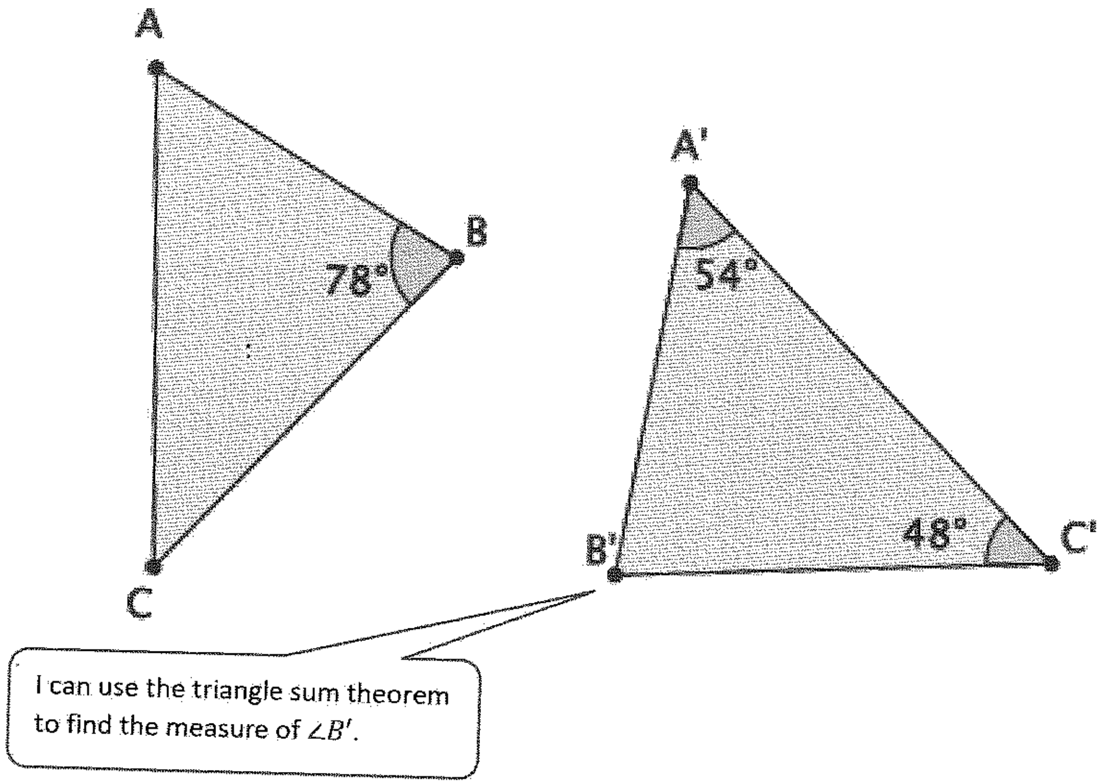
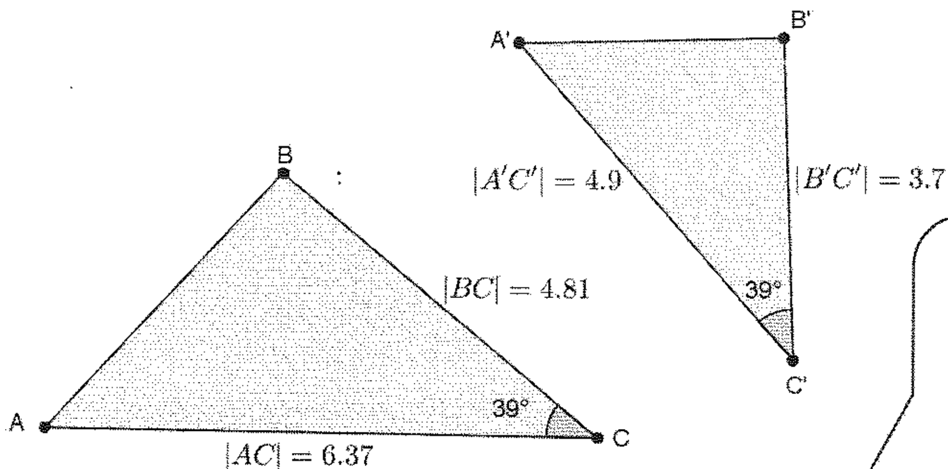
3. Are the triangles shown below similar? Present an informal argument as to why they are or are not similar.



We do not know if $\triangle ABC$ is similar to $\triangle A'B'C'$. We can use the triangle sum theorem to find out that $|\angle B'| = 78°$, but we do not have any information about $|\angle A|$ or $|\angle C|$. To be considered similar, the two triangles must have two pairs of corresponding angles that are equal in measure. In this problem, we only know the measures of one pair of corresponding angles.

G8-M3-Lesson 11: More About Similar Triangles

1. In the diagram below, you have $\triangle ABC$ and $\triangle A'B'C'$. Use this information to answer parts (a)–(b).



I don't have enough information to use the AA criterion. I need to check the ratios of corresponding sides to see if they are equal.

a. Based on the information given, is $\triangle ABC \sim \triangle A'B'C'$? Explain.

Yes, $\triangle ABC \sim \triangle A'B'C'$. Since there is only information about one pair of corresponding angles being equal in measure, then the corresponding sides must be checked to see if their ratios are equal.

$$\frac{4.81}{6.37} = \frac{3.7}{4.9}$$
$$0.755\ldots = 0.755\ldots$$

Since the values of these ratios are equal, approximately 0.755, the triangles are similar.

After I set up the ratio, I need to find the value of x that makes the fractions equivalent.

b. Assume the length of side $\overline{AB}$ is 4.03. What is the length of side $\overline{A'B'}$?

Let x represent the length of side $\overline{A'B'}$.

$$\frac{x}{4.03} = \frac{3.7}{4.81}$$

We are looking for the value of x that makes the fractions equivalent. Therefore, $4.81x = 14.911$, and $x = 3.1$. The length of side $\overline{A'B'}$ is 3.1.

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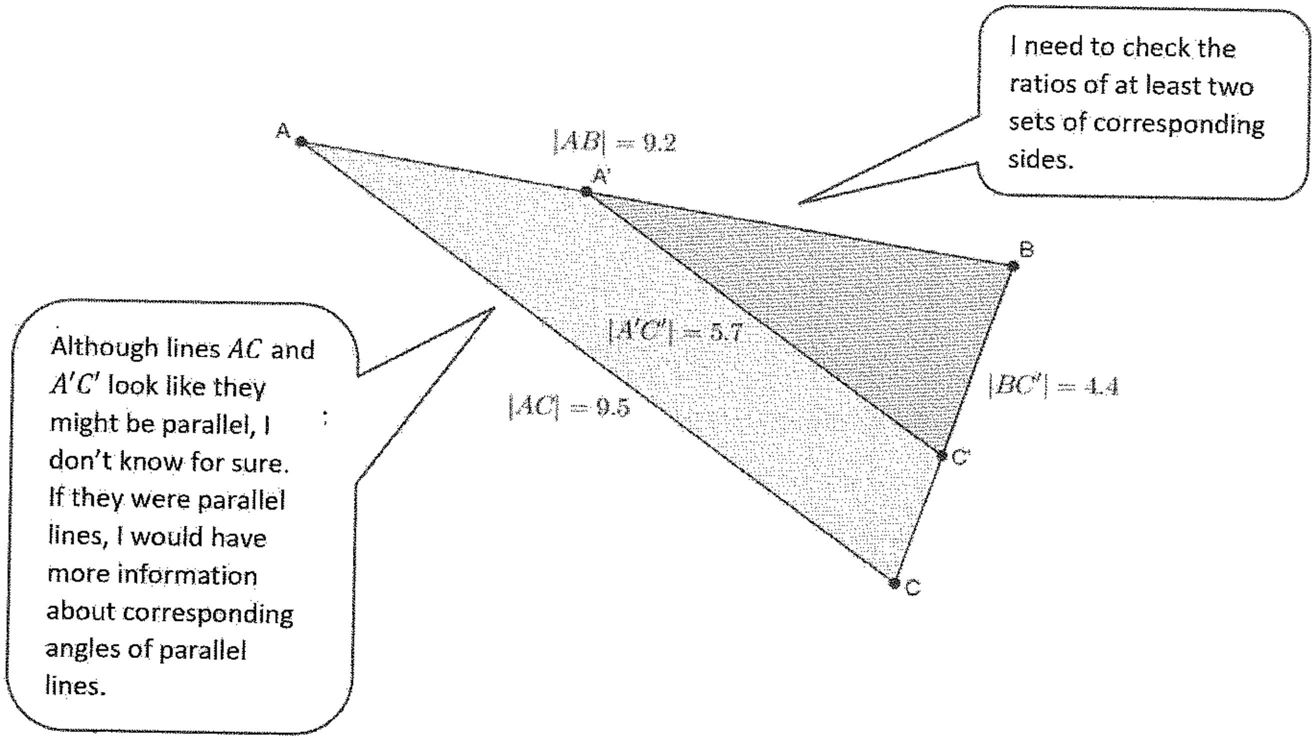
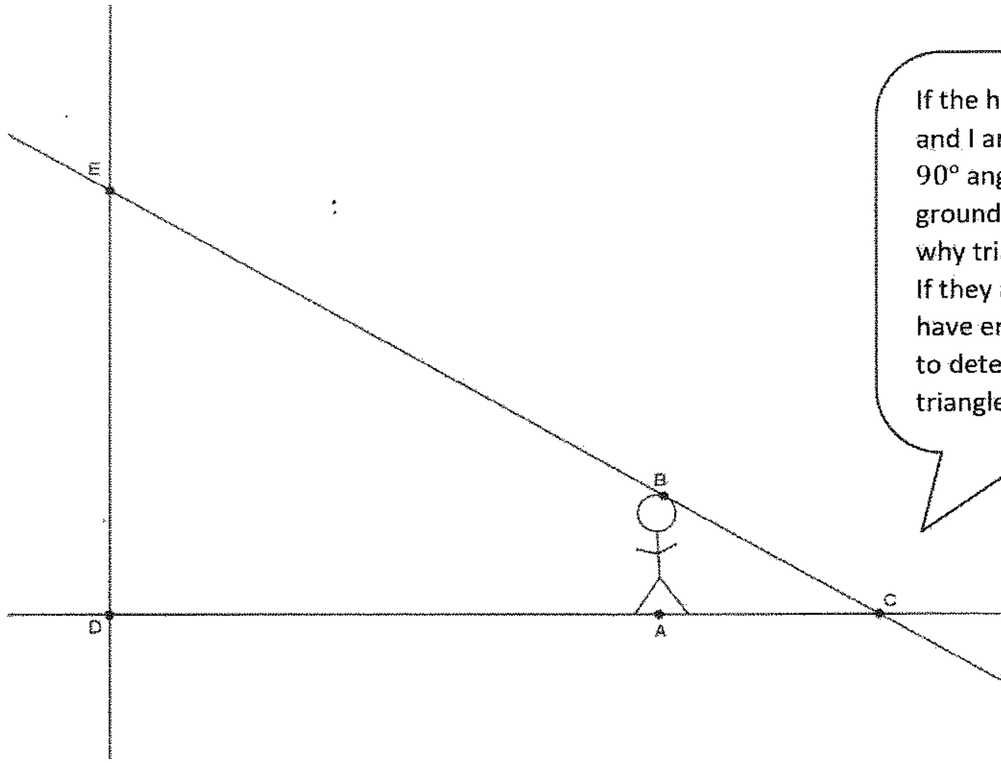
2. In the diagram below, you have $\triangle ABC$ and $\triangle A'BC'$. Based on the information given, is $\triangle ABC \sim \triangle A'BC'$? Explain.



Since both triangles have a common vertex, then $|\angle B| = |\angle B|$. This means that the measure of $\angle B$ in $\triangle ABC$ is equal to the measure of $\angle B$ in $\triangle A'BC'$. However, there is not enough information provided to determine if the triangles are similar. We would need information about a pair of corresponding angles or more information about the side lengths of each of the triangles.

G8-M3-Lesson 12: Modeling Using Similarity

1. There is a statue of your school's mascot in front of your school. You want to find out how tall the statue is, but it is too tall to measure directly. The following diagram represents the situation.



If the height of the statue and I are measured at a 90° angle with the ground, then I can explain why triangles are similar. If they are not, I do not have enough information to determine if the triangles are similar.

Describe the triangles in the situation, and explain how you know whether or not they are similar.

There are two triangles in the diagram, one formed by the statue and the shadow it casts, $\triangle EDC$, and another formed by the person and his shadow, $\triangle BAC$. The triangles are similar if the height of the statue is measured at a 90° angle with the ground and if the person standing forms a 90° angle with the ground. We know that $\angle ACB$ is an angle common to both triangles. If $|\angle EDC| = |\angle BAC| = 90^\circ$, then $\triangle EDC \sim \triangle BAC$ by the AA criterion.

2. Assume $\triangle EDC \sim \triangle BAC$. If the statue casts a shadow 18 feet long and you are 5 feet tall and cast a shadow of 7 feet, find the height of the statue.

Let x represent the height of the statue; then

$$\frac{x}{5} = \frac{18}{7}.$$

Since I know the triangles are similar, I can set up a ratio of corresponding sides.

We are looking for the value of x that makes the fractions equivalent. Therefore, $7x = 90$, and $x = \frac{90}{7}$. The statue is about 13 feet tall.