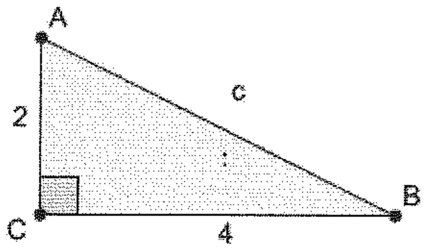
G8-M2-Lesson 15: Informal Proof of the Pythagorean Theorem

For each of the problems below, determine the length of the hypotenuse of the right triangle shown. Note: Figures are not drawn to scale.

1.

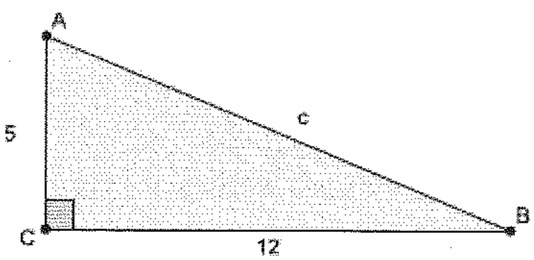


$$a^2 + b^2 = c^2$$
$$2^2 + 4^2 = c^2$$
$$4 + 16 = c^2$$
$$20 = c^2$$

I know that 2 and 4 are the legs of the triangle. I know this because the hypotenuse is across from the $90°$ angle. Since the hypotenuse is side c in my formula, I substitute the 2 and 4 for a and b.

Since I do not know what number times itself produces 20, for now I can leave my answer as $20 = c^2$.

2.

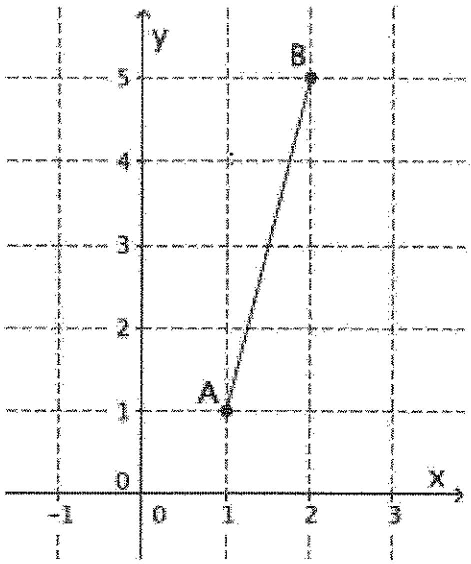


$$a^2 + b^2 = c^2$$
$$12^2 + 5^2 = c^2$$
$$144 + 25 = c^2$$
$$169 = c^2$$
$$13 = c$$

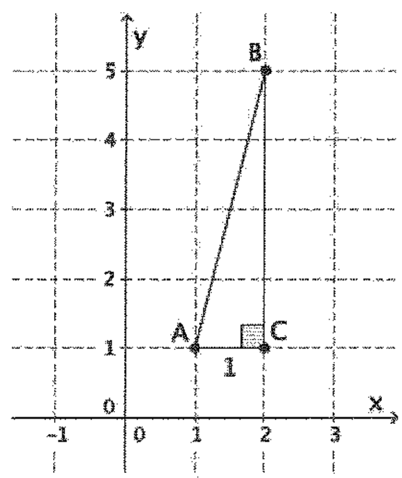
Since I know that $13 \times 13 = 169$, then I know that $c = 13$.

G8-M2-Lesson 16: Applications of the Pythagorean Theorem

1. Find the length of the segment AB shown below, if possible.



I know that the grid lines on the coordinate plane meet at a right angle. I can make my right triangle using a horizontal line through point A and a vertical line through point B.



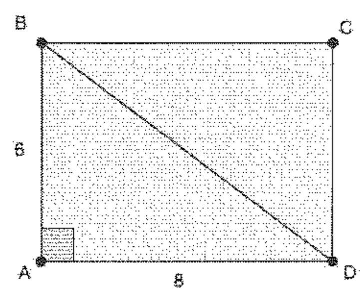
$$a^2 + b^2 = c^2$$
$$1^2 + 4^2 = c^2$$
$$1 + 16 = c^2$$
$$17 = c^2$$

2. A rectangle has dimensions 6 cm by 8 cm. What is the length of the diagonal of the rectangle?

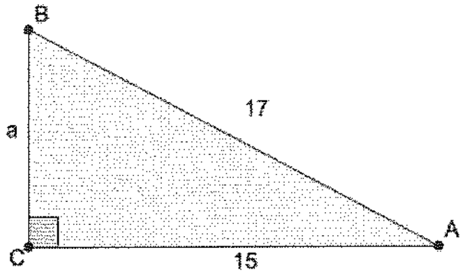
$$a^2 + b^2 = c^2$$
$$6^2 + 8^2 = c^2$$
$$36 + 64 = c^2$$
$$100 = c^2$$
$$10 = c$$

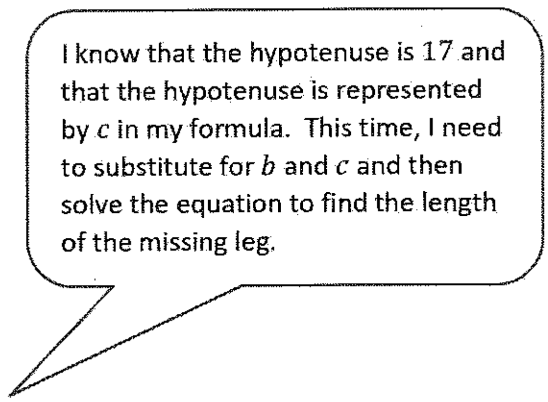
The length of the diagonal is 10 cm.

I should draw the rectangle so I can see the right triangle.



3. Determine the length of the unknown side, if possible.





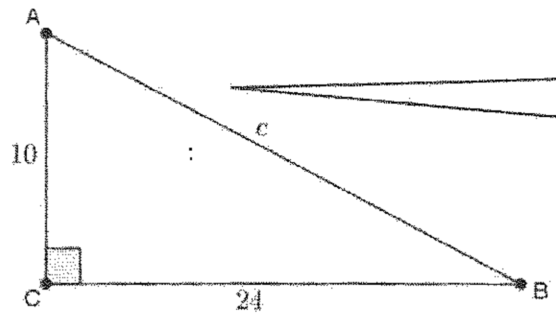
$$a^2 + b^2 = c^2$$
$$a^2 + 15^2 = 17^2$$
$$a^2 + 225 = 289$$
$$a^2 + 225 - 225 = 289 - 225$$
$$a^2 = 64$$
$$a = 8$$

EUREKA MATH™

G8-M3-Lesson 13: Proof of the Pythagorean Theorem

Use the Pythagorean theorem to determine the unknown length of the right triangle.

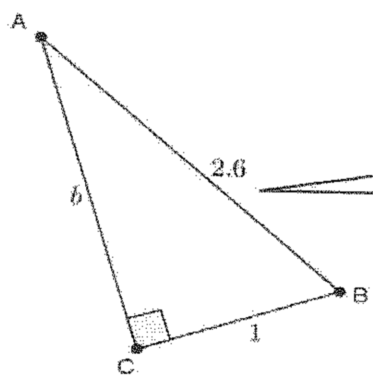
1. Determine the length of side c in the triangle below.



I know the missing side is the hypotenuse because it is the longest side of the triangle and opposite the right angle.

$$10^2 + 24^2 = c^2$$
$$100 + 576 = c^2$$
$$676 = c^2$$
$$26 = c$$

2. Determine the length of side b in the triangle below.

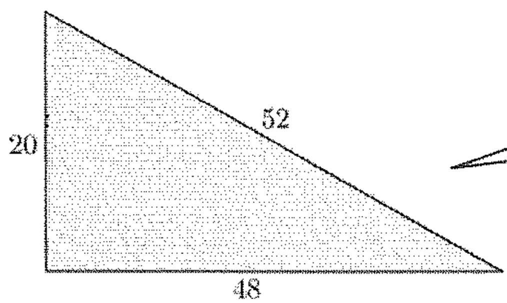


The side lengths are a tenth of the side lengths in problem one. I can multiply the side lengths by ten to make them whole numbers. That will make the problem easier.

$$1^2 + b^2 = 2.6^2$$
$$1 + b^2 = 6.76$$
$$1 - 1 + b^2 = 6.76 - 1$$
$$b^2 = 5.76$$
$$b = 2.4$$

G8-M3-Lesson 14: The Converse of the Pythagorean Theorem

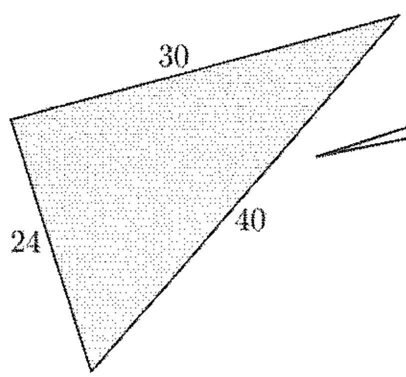
1. The numbers in the diagram below indicate the units of length of each side of the triangle. Is the triangle shown below a right triangle? Show your work, and answer in a complete sentence.



> I need to check to see if $a^2 + b^2 = c^2$ is true. If it is true, then it is right triangle.

We need to check if $20^2 + 48^2 = 52^2$ is a true statement. The left side of the equation is equal to $2,704$. The right side of the equation is equal to $2,704$. That means $20^2 + 48^2 = 52^2$ is true, and the triangle shown is a right triangle.

2. The numbers in the diagram below indicate the units of length of each side of the triangle. Is the triangle shown below a right triangle? Show your work, and answer in a complete sentence.



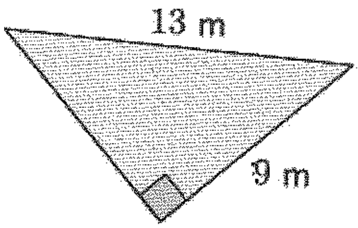
> I remember that c is the longest side of the triangle.

We need to check if $24^2 + 30^2 = 40^2$ is a true statement. The left side of the equation is equal to $1,476$. The right side of the equation is equal to $1,600$. That means $24^2 + 30^2 = 40^2$ is not true, and the triangle shown is not a right triangle.

G8-M7-Lesson 1: The Pythagorean Theorem

1. Use the Pythagorean theorem to estimate the length of the unknown side of the right triangle. Explain why your estimate makes sense.

> I remember the Pythagorean theorem from previous lessons.



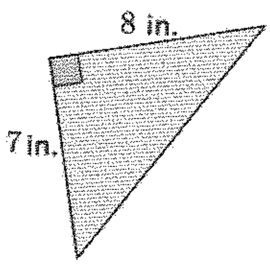
Let x m represent the length of the unknown side.

$$9^2 + x^2 = 13^2$$
$$81 + x^2 = 169$$
$$x^2 = 88$$

> The number 88 is not a perfect square, but I do know that 81 and 100 are perfect squares, and 88 is between them.

The number 88 is between the perfect squares 81 and 100. Since 88 is closer to 81 than it is to 100, the length of the unknown side of the triangle is closer to 9 m than it is to 10 m.

2. Use the Pythagorean theorem to estimate the length of the unknown side of the right triangle. Explain why your estimate makes sense.

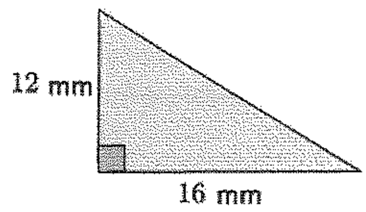


Let c in. represent the length of the hypotenuse.

$$7^2 + 8^2 = c^2$$
$$49 + 64 = c^2$$
$$113 = c^2$$

The number 113 is between the perfect squares 100 and 121. Since 113 is closer to 121 than it is to 100, the length of the hypotenuse of the triangle is closer to 11 in. than it is to 10 in.

3. Use the Pythagorean theorem to estimate the length of the unknown side of the right triangle. Explain why your estimate makes sense.



Let c mm represent the length of the hypotenuse.

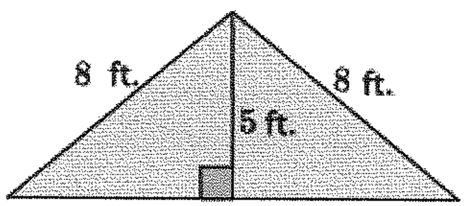
$$12^2 + 16^2 = c^2$$
$$144 + 256 = c^2$$
$$400 = c^2$$
$$20 = c$$

The number 400 is a perfect square, so I know that 20 is equal to c.

The length of the hypotenuse is 20 mm. The Pythagorean theorem led me to the fact that the square of the unknown side is 400. We know 400 is a perfect square, and 400 is equal to 20^2; therefore, $c = 20$, and the length of the hypotenuse of the triangle is 20 mm.

4. The triangle below is an isosceles triangle. Use what you know about the Pythagorean theorem to determine the approximate length of the base of the isosceles triangle.

I can find the length of the base of one triangle and multiply that number by 2 since the two right triangles are congruent.

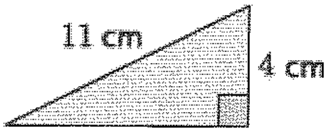


Let x ft. represent the length of the base of one of the right triangles of the isosceles triangle.

$$x^2 + 5^2 = 8^2$$
$$x^2 + 25 = 64$$
$$x^2 = 39$$

Since 39 is between the perfect squares 36 and 49 but closer to 36, the approximate length of the base of the right triangle is 6 ft. Since there are two right triangles, the length of the base of the isosceles triangle is approximately 12 ft.

5. Give an estimate for the area of the triangle shown below. Explain why it is a good estimate.



Let x cm represent the length of the base of the right triangle.

$$x^2 + 4^2 = 11^2$$
$$x^2 + 16 = 121$$
$$x^2 = 105$$

I need to use the base, b, and height, h, of the triangle to find the area, A, of a triangle; $A = \frac{1}{2}bh$.

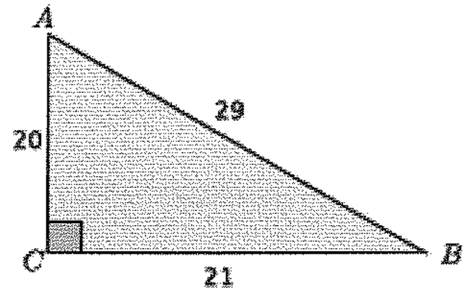
Since 105 is between the perfect squares 100 and 121 but closer to 100, the approximate length of the base is 10 cm. $A = \frac{1}{2}(10)(4) = 20$. *So, the approximate area of the triangle is 20 cm^2.*

The hypotenuse is the longest side, so the base must be less than 11 cm.

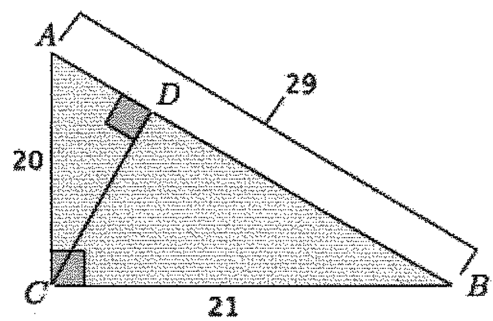
20 cm^2 is a good estimate because of the approximation of the length of the base. Furthermore, since the hypotenuse is the longest side of the right triangle, approximating the length of the base as 10 cm makes mathematical sense because it has to be shorter than the hypotenuse.

G8-M7-Lesson 15: Pythagorean Theorem, Revisited

1. For the right triangle shown below, identify and use similar triangles to illustrate the Pythagorean theorem.

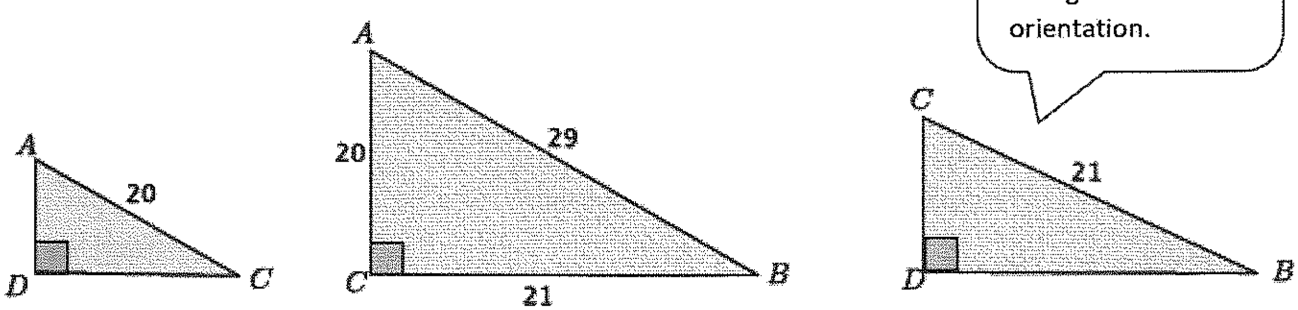


First, draw a segment that is perpendicular to side $\overline{AB}$ that goes through point C. The point of intersection of that segment and side $\overline{AB}$ will be marked as point D.



Then, I have three similar triangles, $\triangle ABC$, $\triangle CBD$, and $\triangle ACD$, as shown below.

Using a sequence of transformations, I can see each right triangle in the same orientation.



$\triangle ABC$ and $\triangle CBD$ are similar because each one has a right angle, and they both share $\angle B$. By AA criterion, $\triangle ABC \sim \triangle CBD$.

$\triangle ABC$ and $\triangle ACD$ are similar because each one has a right angle, and they both share $\angle A$. By AA criterion, $\triangle ABC \sim \triangle ACD$.

By the transitive property, we also know that $\triangle ACD \sim \triangle CBD$.

Since the triangles are similar, they have corresponding sides that are equal in ratio.

For $\triangle ABC$ *and* $\triangle CBD$,

$$\frac{21}{29} = \frac{|BD|}{21},$$

which is the same as $21^2 = 29(|BD|)$.

Because my right triangles are in the same orientation, I can make the following proportion using $\triangle ABC$ and $\triangle CBD$: $\frac{\text{base}}{\text{hypotenuse}} = \frac{\text{base}}{\text{hypotenuse}}$. I can solve any proportion by using cross multiplication.

For $\triangle ABC$ *and* $\triangle ACD$,

$$\frac{20}{29} = \frac{|AD|}{20},$$

which is the same as $20^2 = 29(|AD|)$.

Adding these two equations together, I get

$$21^2 + 20^2 = 29(|BD|) + 29(|AD|).$$

By the distributive property,

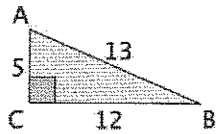
$$21^2 + 20^2 = 29(|BD| + |AD|);$$

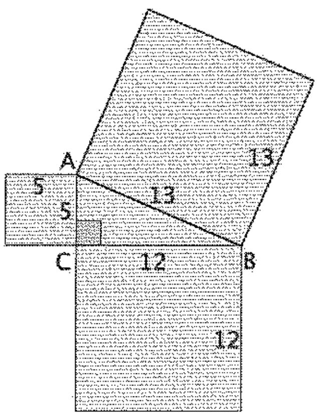
however, $|BD| + |AD| = |AB| = 29$. *Therefore,*

I found this by looking at my original triangle and the triangle where the altitude was drawn.

$$20^2 + 21^2 = 29(29)$$
$$20^2 + 21^2 = 29^2.$$

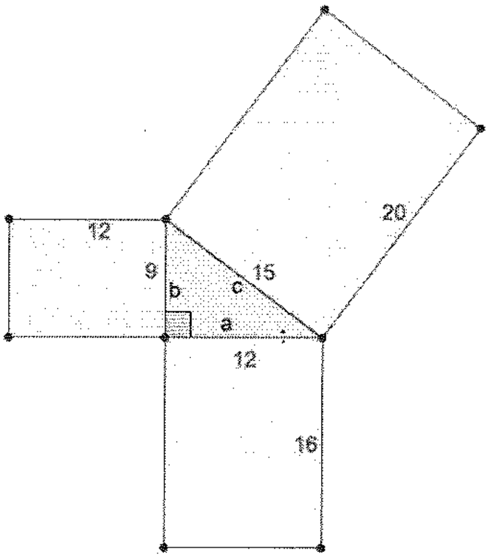
2. For the right triangle shown below, identify and use squares formed by the sides of the triangle to illustrate the Pythagorean theorem.





The sum of the areas of the smallest squares is $5^2 + 12^2$, *which is* 169. *The area of the largest square is* 13^2, *which is* 169. *The sum of the areas of the squares of the legs is equal to the area of the square of the hypotenuse; therefore, for legs* a *and* b, *and hypotenuse* c, *we see that* $a^2 + b^2 = c^2$.

3. Can any similar figures be drawn off the sides of the right triangle to prove the Pythagorean theorem? Use computations to show that the sum of the areas of the figures off of the sides a and b equals the area of the figure off of side c.



The rectangles are similar because their corresponding side lengths are equal in scale factor.

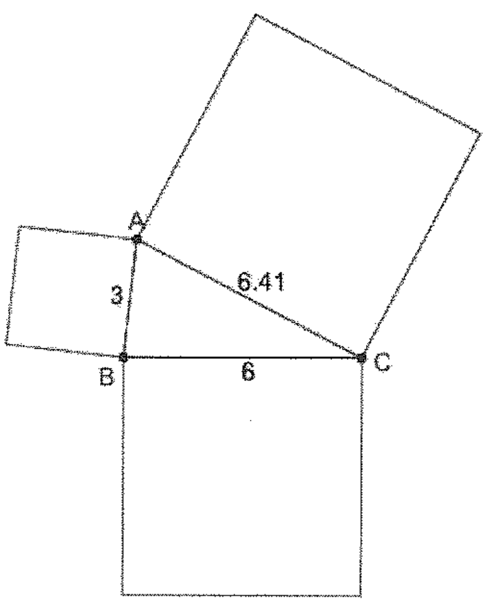
$$\frac{9}{12} = \frac{12}{16} = \frac{15}{20} = 0.75$$

The area of the smaller rectangles are 108 square units and 192 square units, and the area of the larger rectangle is 300 square units. The sum of the smaller areas is equal to the larger area.

$$108 + 192 = 300$$
$$300 = 300$$

Therefore, the sum of the areas of the smaller similar rectangles does equal the area of the larger similar rectangle proving the Pythagorean theorem with similar figures.

4. The following image for the Pythagorean theorem contains an error. Explain what is wrong.



Based on the proof shown in class, we would expect the sum of the two smaller areas to be equal to the larger area. The smaller areas are 9 and 36, while the larger area is 41.0881. That is, 9 + 36 should equal 41.0881. However, 9 + 36 = 45.

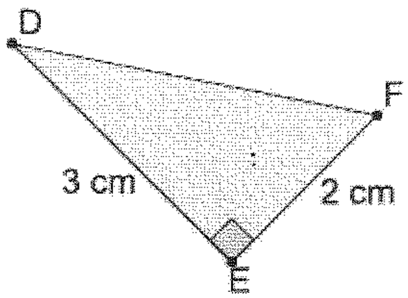
We know that the Pythagorean theorem only works for right triangles. Considering the converse of the Pythagorean theorem, when we use the given side lengths, we do not get a true statement.

$$3^2 + 6^2 = 6.41^2$$
$$9 + 36 = 41.0881$$
$$45 \neq 41.0881$$

Therefore, the original triangle is not a right triangle, so it makes sense that the areas of the squares were not adding up like we expected.

G8-M7-Lesson 16: Converse of the Pythagorean Theorem

1. What is the length of the unknown side of the right triangle shown below? Show your work, and answer in a complete sentence. Provide an exact answer and an approximate answer rounded to the tenths place.



Let c cm represent the hypotenuse of the triangle.

$$3^2 + 2^2 = c^2$$
$$9 + 4 = c^2$$
$$13 = c^2$$
$$\sqrt{13} = \sqrt{c^2}$$
$$3.6 \approx c$$

To estimate, I need the two perfect squares that surround 13, which are 9 and 16. 13 is slightly closer to 16 than 9, so $\sqrt{13}$ is closer to $\sqrt{16} = 4$ than $\sqrt{9} = 3$.

The hypotenuse of the triangle is exactly $\sqrt{13}$ cm and approximately 3.6 cm.

2. Is the triangle with leg lengths of $\sqrt{5}$ cm and 7 cm and hypotenuse of length $\sqrt{54}$ cm a right triangle? Show your work, and answer in a complete sentence.

$$\left(\sqrt{5}\right)^2 + 7^2 = \left(\sqrt{54}\right)^2$$
$$5 + 49 = 54$$
$$54 = 54$$

To simplify a square root that is squared, I need to remember the following:

$$\left(\sqrt{5}\right)^2 = \sqrt{5} \cdot \sqrt{5}$$
$$\left(\sqrt{5}\right)^2 = \sqrt{25}$$
$$\left(\sqrt{5}\right)^2 = 5$$

Yes, the triangle with leg lengths of $\sqrt{5}$ cm and 7 cm and hypotenuse of length $\sqrt{54}$ cm is a right triangle because the lengths satisfy the Pythagorean theorem.

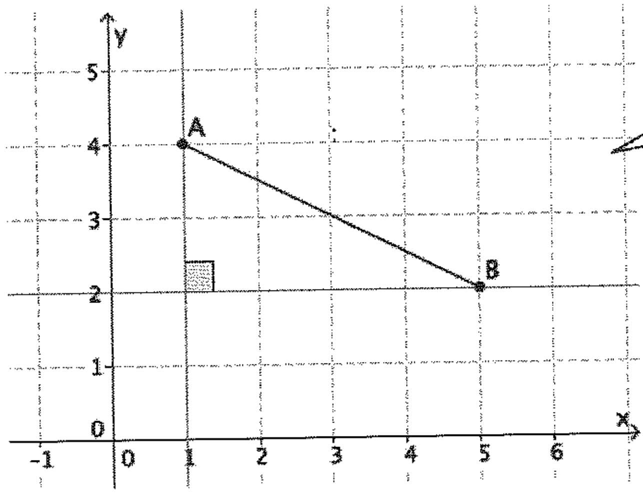
3. Is the triangle with leg lengths of $\sqrt{8}$ cm and 10 cm and hypotenuse of length $\sqrt{164}$ cm a right triangle? Show your work, and answer in a complete sentence.

$$\left(\sqrt{8}\right)^2 + 10^2 = \left(\sqrt{164}\right)^2$$
$$8 + 100 = 164$$
$$108 \neq 164$$

No, the triangle with leg lengths of $\sqrt{8}$ cm and 10 cm and hypotenuse of length $\sqrt{164}$ cm is not a right triangle because the lengths do not satisfy the Pythagorean theorem.

G8-M7-Lesson 17: Distance on the Coordinate Plane

1. Determine the distance between points A and B on the coordinate plane. Round your answer to the tenths place.



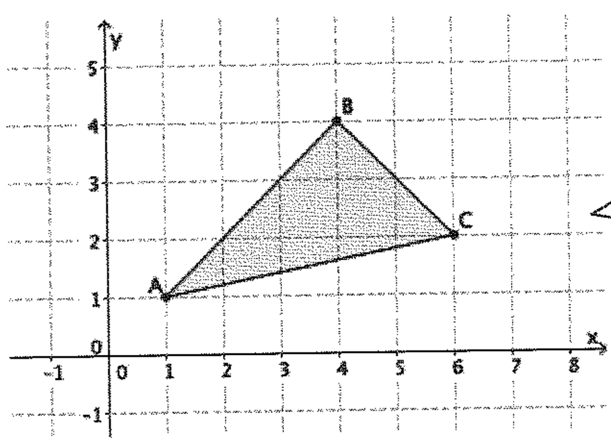
To determine the distance between the points, I am going to use the grid lines from the coordinate plane to create a right triangle.

Let c represent $|AB|$.

$$2^2 + 4^2 = c^2$$
$$4 + 16 = c^2$$
$$20 = c^2$$
$$\sqrt{20} = c$$
$$4.5 \approx c$$

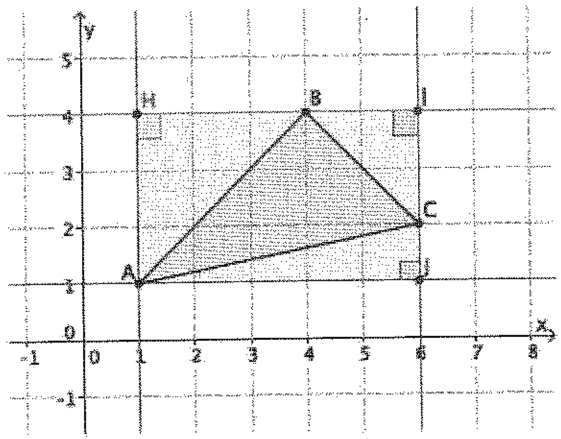
The distance between points A and B is about 4.5 units.

2. Is the triangle formed by points A, B, and C a right triangle?



None of these sides are horizontal or vertical, so I cannot simply count to find their lengths.

I need to create right triangles to find the lengths of the sides and then use those lengths in the Pythagorean theorem.



Let c represent $|AB|$.

$$3^2 + 3^2 = c^2$$
$$9 + 9 = c^2$$
$$18 = c^2$$
$$\sqrt{18} = \sqrt{c^2}$$
$$\sqrt{18} = c$$

Let c represent $|BC|$.

$$2^2 + 2^2 = c^2$$
$$4 + 4 = c^2$$
$$8 = c^2$$
$$\sqrt{8} = \sqrt{c^2}$$
$$\sqrt{8} = c$$

Let c represent $|AC|$.

$$5^2 + 1^2 = c^2$$
$$25 + 1 = c^2$$
$$26 = c^2$$
$$\sqrt{26} = \sqrt{c^2}$$
$$\sqrt{26} = c$$

The hypotenuse is side $\overline{AC}$ since $\sqrt{26}$ is the longest side. I substituted into the Pythagorean theorem and got a true statement, meaning I have a right triangle.

$$\left(\sqrt{18}\right)^2 + \left(\sqrt{8}\right)^2 = \left(\sqrt{26}\right)^2$$
$$18 + 8 = 26$$
$$26 = 26$$

Yes, the points do form a right triangle.

G8-M7-Lesson 18: Applications of the Pythagorean Theorem

1. A 55 in. TV is advertised on sale at a local store. What are the length and width of the television?

Let x be the factor applied to the ratio $4:3$.

$$(4x)^2 + (3x)^2 = 55^2$$
$$16x^2 + 9x^2 = 3025$$
$$25x^2 = 3025$$
$$\frac{25}{25}x^2 = \frac{3025}{25}$$
$$x^2 = 121$$
$$\sqrt{x^2} = \sqrt{121}$$
$$x = 11$$

I remember from the notes that the dimensions of a television are in the ratio $4:3$ and that the size of the television is actually the length of the diagonal. Therefore, $4x$ and $3x$ are the legs of the right triangle while 55 is the hypotenuse.

Since $x = 11$, $3x = 33$ and $4x = 44$. Therefore, the dimensions of the TV are 44 in. by 33 in.

2. The soccer team was instructed to run the perimeter of their soccer field, which has dimensions of 115 yards by 74 yards. One player decided to run the length and width and then finished by running diagonally. To the nearest tenth of a yard, how many fewer yards of running did this player complete than the rest of the team?

$$P = 2l + 2w$$
$$P = 2(115) + 2(74)$$
$$P = 230 + 148$$
$$P = 378$$

The team ran 378 yards.

Let a yards represent the length of the field, b yards represent the width of the field, and c yards represent the diagonal length of the field.

$$a^2 + b^2 = c^2$$
$$115^2 + 74^2 = c^2$$
$$13225 + 5476 = c^2$$
$$18701 = c^2$$
$$\sqrt{18701} = \sqrt{c^2}$$
$$136.8 \approx c$$

I need to find the diagonal length of the field and then use it to find the total distance the single player ran. The number $\sqrt{18{,}701}$ is between 136 and 137. In the sequence of tenths, the number is between 136.7 and 136.8 because $136.7^2 < \left(\sqrt{18{,}701}\right)^2 < 136.8^2$. Since 18,701 is closer to 136.8^2 than 136.7^2, the approximate length of the hypotenuse is 136.8 yards.

$$115 + 74 + 136.8 = 325.8$$

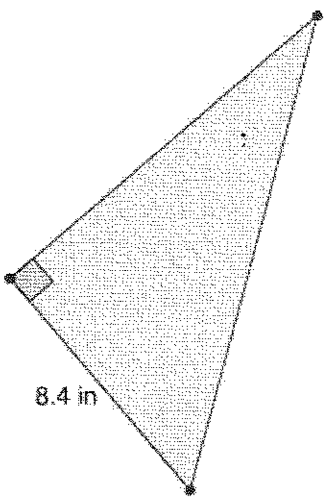
The player ran approximately 325.8 *yards.*

$$378 - 325.8 = 52.2$$

The player ran approximately 52.2 *fewer yards than the rest of the team.*

3. The area of the right triangle shown below is $51.24\ \text{in}^2$.

   a. What is the height of the triangle?



Let h in. *represent the height of the triangle.*

$$A = \frac{1}{2}bh$$
$$51.24 = \frac{1}{2}(8.4)h$$
$$51.24 = 4.2h$$
$$\frac{51.24}{4.2} = \frac{4.2}{4.2}h$$
$$12.2 = h$$

The height of the triangle is 12.2 in.

   b. What is the perimeter of the right triangle? Round your answer to the tenths place.

Let c in. *represent the length of the hypotenuse.*

$$8.4^2 + 12.2^2 = c^2$$
$$70.56 + 148.84 = c^2$$
$$219.4 = c^2$$
$$\sqrt{219.4} = \sqrt{c^2}$$
$$14.8 \approx c$$

In order to find the perimeter, I first need all three sides. To find the hypotenuse, I can use the Pythagorean theorem.

The number $\sqrt{219.4}$ *is between* 14 *and* 15. *In the sequence of tenths, the number is between* 14.8 *and* 14.9 *because* $14.8^2 < \left(\sqrt{219.4}\right)^2 < 14.9^2$. *Since* 219.4 *is closer to* 14.8^2 *than* 14.9^2, *the approximate length of the hypotenuse is* 14.8 in.

$$8.4 + 12.2 + 14.8 = 35.4$$

The perimeter of the triangle is approximately 35.4 in.