Eureka Math™ Homework Helper

2015–2016

Grade 8
Module 1
Lessons 1–13

Eureka Math, A Story of Ratios®

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G8-M1-Lesson 1: Exponential Notation

Use what you know about exponential notation to complete the expressions below.

1. $\underbrace{(-2) \times \cdots \times (-2)}_{35 \text{ times}} = \mathbf{(-2)^{35}}$

2. $\underbrace{\left(\frac{9}{2}\right) \times \cdots \times \left(\frac{9}{2}\right)}_{12 \text{ times}} = \mathbf{\left(\frac{9}{2}\right)^{12}}$

When the base (the number being repeatedly multiplied) is negative or fractional, I need to use parentheses. If I don't, the number being multiplied won't be clear. Some may think that the 2 or only the numerator of the fraction gets multiplied.

3. $\underbrace{8 \times \cdots \times 8}_{\_\_\_\_ \text{ times}} = 8^{56}$

The exponent states how many times the 8 is multiplied. It is multiplied 56 times so that is what is written in the blank.

4. Rewrite each number in exponential notation using 3 as the base.
   a. $9 = \mathbf{3 \times 3 = 3^2}$
   b. $27 = \mathbf{3 \times 3 \times 3 = 3^3}$
   c. $81 = \mathbf{3 \times 3 \times 3 \times 3 = 3^4}$
   d. $243 = \mathbf{3 \times 3 \times 3 \times 3 \times 3 = 3^5}$

All I need to do is figure out how many times to multiply 3 in order to get the number I'm looking for in parts (a)–(d).

5. Write an expression with (-2) as its base that will produce a negative product.

One possible solution is shown below.

$\mathbf{(-2)^3 = (-2) \times (-2) \times (-2) = -8}$

To produce a negative product, I need to make sure the negative number is multiplied an odd number of times. Since the product of two negative numbers results in a positive product, multiplying one more time will result in a negative product.

G8-M1-Lesson 2: Multiplication of Numbers in Exponential Form

Let x, a, and b be numbers and $b \neq 0$. Write each expression using the fewest number of bases possible.

1. $(-7)^3 \cdot (-7)^4 =$

   $\boldsymbol{(-7)^{3+4}}$

I have to be sure that the base of each term is the same if I intend to use the identity $x^m \cdot x^n = x^{m+n}$ for Problems 1–4.

2. $x^5 \cdot x^6 =$

   $\boldsymbol{x^{5+6}}$

3. $\left(\frac{2}{3}\right)^7 \cdot \left(\frac{2}{3}\right)^5 =$

   $\boldsymbol{\left(\frac{2}{3}\right)^{7+5}}$

4. $2^4 \cdot 8^2 =$

   $\boldsymbol{2^4 \cdot (2^3)^2 =}$

   $\boldsymbol{2^4 \cdot 2^3 \cdot 2^3 =}$

   $\boldsymbol{2^{4+3+3}}$

For this problem, I know that $8 = 2^3$, so I can transform the 8 to have a base of 2.

5. $ab^3 \cdot a^4b^5 =$

   $\boldsymbol{a \cdot a^4 \cdot b^3 \cdot b^5 =}$

   $\boldsymbol{a^{1+4} \cdot b^{3+5}}$

In this problem, I see two bases, a and b. I can use the commutative property to reorder the a's so that they are together and reorder the b's so they are together.

6. $\frac{(-3)^5}{(-3)^2} =$

   $\boldsymbol{(-3)^{5-2}}$

When the bases are the same for division problems, I can use the identity $\frac{x^m}{x^n} = x^{m-n}$.

7. $\frac{a^2b^5}{b^2} =$

   $\boldsymbol{a^2 \cdot \frac{b^5}{b^2} =}$

   $\boldsymbol{a^2b^{5-2}}$

8. $\frac{27}{3^2} =$

   $\boldsymbol{\frac{3^3}{3^2} =}$

   $\boldsymbol{3^{3-2}}$

The number 27 is the same as $3 \times 3 \times 3$ or 3^3.

G8-M1-Lesson 3: Numbers in Exponential Form Raised to a Power

Lesson Notes

Students will be able to rewrite expressions involving powers to powers and products to powers. The following two identities will be used:

For any number x and any positive integers m and n, $(x^m)^n = x^{mn}$.

For any numbers x and y and positive integer n, $(xy)^n = x^n y^n$.

In the lesson today, we learned to use the identity to simplify these expressions. If the directions say show in detail, or prove, I know I need to use the identities and properties I knew before this lesson to show that the identity I learned today actually holds true.

Show (prove) in detail why $(3 \cdot x \cdot y)^5 = 3^5 \cdot x^5 \cdot y^5$.

$(\mathbf{3} \cdot \boldsymbol{x} \cdot \boldsymbol{y})^{\mathbf{5}} = (\mathbf{3} \cdot \boldsymbol{x} \cdot \boldsymbol{y}) \cdot (\mathbf{3} \cdot \boldsymbol{x} \cdot \boldsymbol{y}) \cdot (\mathbf{3} \cdot \boldsymbol{x} \cdot \boldsymbol{y}) \cdot (\mathbf{3} \cdot \boldsymbol{x} \cdot \boldsymbol{y}) \cdot (\mathbf{3} \cdot \boldsymbol{x} \cdot \boldsymbol{y})$ ***By the definition of exponential notation***

$= (\mathbf{3} \cdot \mathbf{3} \cdot \mathbf{3} \cdot \mathbf{3} \cdot \mathbf{3}) \cdot (\boldsymbol{x} \cdot \boldsymbol{x} \cdot \boldsymbol{x} \cdot \boldsymbol{x} \cdot \boldsymbol{x}) \cdot (\boldsymbol{y} \cdot \boldsymbol{y} \cdot \boldsymbol{y} \cdot \boldsymbol{y} \cdot \boldsymbol{y})$ ***By the commutative and associative properties***

$= \mathbf{3}^{\mathbf{5}} \cdot \boldsymbol{x}^{\mathbf{5}} \cdot \boldsymbol{y}^{\mathbf{5}}$ ***By the definition of exponential notation or by the first law of exponents***

$= \mathbf{3}^{\mathbf{5}}\boldsymbol{x}^{\mathbf{5}}\boldsymbol{y}^{\mathbf{5}}$

If I am going to use the first law of exponents to explain this part of my proof, I might want to show another line in my work that looks like this: $3^{1+1+1+1+1} \cdot x^{1+1+1+1+1} \cdot y^{1+1+1+1+1}$.

G8-M1-Lesson 4: Numbers Raised to the Zeroth Power

Let x, y, f, and g be numbers ($x, y, f, g \neq 0$). Simplify each of the following expressions.

1. $\frac{x^6}{x^6}$

$= \boldsymbol{x^{6-6}}$
$= \boldsymbol{x^0}$
$= \boldsymbol{1}$

In class, I know we defined a number raised to the zero power as 1.

2. $\frac{x^3y^4}{x^3y^4}$

$= \boldsymbol{x^{3-3}y^{4-4}}$
$= \boldsymbol{x^0y^0}$
$= \boldsymbol{1 \cdot 1}$
$= \boldsymbol{1}$

3. $4^5 \cdot \frac{1}{4^5}$

$= \boldsymbol{\frac{4^5}{4^5}}$
$= \boldsymbol{4^{5-5}}$
$= \boldsymbol{4^0}$
$= \boldsymbol{1}$

I have to use the rule for multiplying fractions.

I multiply the numerator times the numerator and the denominator times the denominator.

4. $3^7 \cdot \frac{1}{3^5} \cdot 3^5 \cdot \frac{1}{3^7} \cdot 3^2 \cdot \frac{1}{3^2}$

$= \boldsymbol{\frac{3^7 \cdot 1 \cdot 3^5 \cdot 1 \cdot 3^2 \cdot 1}{3^5 \cdot 3^7 \cdot 3^2}}$
$= \boldsymbol{\frac{3^{7+5+2}}{3^{5+7+2}}}$
$= \boldsymbol{\frac{3^{14}}{3^{14}}}$
$= \boldsymbol{3^{14-14}}$
$= \boldsymbol{3^0}$
$= \boldsymbol{1}$

5. $\frac{f^4 \cdot g^3}{g^3 \cdot f^4}$

$= \boldsymbol{\frac{f^4 \cdot g^3}{f^4 \cdot g^3}}$
$= \boldsymbol{f^{4-4} \cdot g^{3-3}}$
$= \boldsymbol{f^0g^0}$
$= \boldsymbol{1 \cdot 1}$
$= \boldsymbol{1}$

6. $\left(8^2(2^6)\right)^0$

$= \boldsymbol{(8^2)^0 \cdot (2^6)^0}$
$= \boldsymbol{8^{2 \cdot 0} \cdot 2^{6 \cdot 0}}$
$= \boldsymbol{8^0 \cdot 2^0}$
$= \boldsymbol{1 \cdot 1}$
$= \boldsymbol{1}$

There is a power outside of the grouping symbol. That means I must use the third and second laws of exponents to simplify this expression.

G8-M1-Lesson 5: Negative Exponents and the Laws of Exponents

Lesson Notes

You will need your Equation Reference Sheet. The numbers in parentheses in the solutions below correlate to the reference sheet.

Examples

1. Compute: $(-2)^4 \cdot (-2)^3 \cdot (-2)^{-2} \cdot (-2)^0 \cdot (-2)^{-2}$

$$= (-2)^{4+3+(-2)+0+(-2)}$$
$$= (-2)^3$$
$$= -8$$

Negative exponents follow the same identities as positive exponents and exponents of zero.

2. Without using (10), show directly that $(y^{-1})^6 = y^{-6}$.

$(y^{-1})^6 = \left(\frac{1}{y^1}\right)^6$ — ***By definition of negative exponents* (9)**

$= \frac{1^6}{y^6}$ — **By $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$ (14)**

$= \frac{1}{y^6}$

$= y^{-6}$ — ***By definition of negative exponents* (9)**

3. Without using (13), show directly that $\frac{6^{-9}}{6^3} = 6^{-12}$.

$\frac{6^{-9}}{6^3} = 6^{-9} \cdot \frac{1}{6^3}$ — ***By product formula for complex fractions***

$= \frac{1}{6^9} \cdot \frac{1}{6^3}$ — ***By definition of negative exponents* (9)**

$= \frac{1}{6^9 \cdot 6^3}$ — ***By product formula for complex fractions***

$= \frac{1}{6^{9+3}}$ — **By $x^m \cdot x^n = x^{m+n}$ (10)**

$= \frac{1}{6^{12}}$

$= 6^{-12}$ — ***By definition of negative exponents* (9)**

G8-M1-Lesson 6: Proofs of Laws of Exponents

Lesson Notes

You will need your Equation Reference Sheet. The numbers in parentheses in the solutions below correlate to the reference sheet.

Examples

1. A very contagious strain of bacteria was contracted by two people who recently travelled overseas. When the couple returned, they then infected three people. The next week, each of those three people infected three more people. This infection rate continues each week. By the end of 5 weeks, how many people would be infected?

The 3 people infected upon return each infect 3 people.

Therefore, in week 1, there are 9 new infected people, or $(3 \times 3) = 3^2$.

Those 9 people infect 3 people each, or 27 new people. $(3^2 \times 3) = 3^3$

Week of Return	$\mathbf{2+3}$
Week 1	$\mathbf{(3 \times 3) + (2+3)}$
Week 2	$\mathbf{(3^2 \times 3) + (3 \times 3) + (2+3)}$
Week 3	$\mathbf{(3^3 \times 3) + (3^2 \times 3) + (3 \times 3) + (2+3)}$
Week 4	$\mathbf{(3^4 \times 3) + (3^3 \times 3) + (3^2 \times 3) + (3 \times 3) + (2+3)}$
Week 5	$\mathbf{(3^5 \times 3) + (3^4 \times 3) + (3^3 \times 3) + (3^2 \times 3) + (3 \times 3) + (2+3)}$

2. Show directly that $r^{-10} \cdot r^{-12} = r^{-22}$.

"Show directly" and "prove" mean the same thing: I should use the identities and definitions I know are true for whole numbers to prove the identities are true for integer exponents also.

$\mathbf{r^{-10} \cdot r^{-12} = \frac{1}{r^{10}} \cdot \frac{1}{r^{12}}}$ ***By definition of negative exponents*** **(9)**

$\mathbf{= \frac{1}{r^{10} \cdot r^{12}}}$ ***By product formula for complex fractions***

$\mathbf{= \frac{1}{r^{10+12}}}$ ***By*** $\boldsymbol{x^m \cdot x^n = x^{m+n}}$ ***for whole numbers*** $\boldsymbol{m}$ ***and*** $\boldsymbol{n}$ **(6)**

$\mathbf{= \frac{1}{r^{22}}}$

$\mathbf{= r^{-22}}$ ***By definition of negative exponents*** **(9)**

G8-M1-Lesson 7: Magnitude

1. What is the smallest power of 10 that would exceed 6,234,579?

   M has 7 digits, so a number with 8 digits will exceed it.

   If I create a number with the same number of digits as M but with all nines, I know that number will exceed M. If I then add 1, I will have a number that can be written as a power of 10.

   $$M = 6,234,579 < 9,999,999 < 10,000,000 = 10^7$$

   The smallest power of 10 that would exceed $6,234,579$ is 10^7.

2. Which number is equivalent to 0.001: 10^3 or 10^{-3}? How do you know?

   10^{-3} is equivalent to 0.001. Positive powers of 10 create large numbers, and negative powers of 10 create numbers smaller than one. The number 10^{-3} is equal to the fraction $\frac{1}{10^3}$, which is the same as $\frac{1}{1000}$ and 0.001. Since 0.001 is a small number, its power of 10 should be negative.

3. Jessica said that 0.0001 is bigger than 0.1 because the first number has more digits to the right of the decimal point. Is Jessica correct? Explain your thinking using negative powers of 10 and the number line.

   $0.0001 = \frac{1}{10000} = 10^{-4}$ and $0.1 = \frac{1}{10} = 10^{-1}$. On a number line 10^{-1} is closer to 0 than 10^{-4}; therefore, 10^{-1} is larger than 10^{-4}.

   I have to remember that negative exponents behave differently than positive exponents. I have to think about the number line and that the further right a number is, the larger the number is.

4. Order the following numbers from least to greatest:

   $10^2 \quad 10^{-4} \quad 10^0 \quad 10^{-3}$

   $\boldsymbol{10^{-4} < 10^{-3} < 10^0 < 10^2}$

   Since all of the bases are the same, I just need to make sure I have the exponents in order from least to greatest.

G8-M1-Lesson 8: Estimating Quantities

1. A 250 gigabyte hard drive has a total of 250,000,000,000 bytes of available storage space. A 3.5 inch double-sided floppy disk widely used in the 1980's could hold about 8×10^5 bytes. How many double-sided floppy disks would it take to fill the 250 gigabyte hard drive?

$$\mathbf{250,000,000,000 \approx 3 \times 10^{11}}$$

$$\mathbf{\frac{3 \times 10^{11}}{8 \times 10^5} = \frac{3}{8} \times \frac{10^{11}}{10^5}}$$
$$\mathbf{= 0.375 \times 10^{11-5}}$$
$$\mathbf{= 0.375 \times 10^6}$$
$$\mathbf{= 375,000}$$

It would take 375,000 floppy disks to fill the 250 gigabyte hard drive.

I know that when the question says, "How many will it take to fill…," it means to divide.

2. A calculation of the operation $2{,}000{,}000 \times 3{,}000{,}000{,}000$ gives an answer of $6\ e + 15$. What does the answer of $6\ e + 15$ on the screen of the calculator mean? Explain how you know.

The answer means 6×10^{15}. This is known because

$$\mathbf{(2 \times 10^6) \times (3 \times 10^9) = (2 \times 3) \times (10^6 \times 10^9)}$$
$$\mathbf{= 6 \times 10^{6+9}}$$
$$\mathbf{= 6 \times 10^{15}}$$

I know that multiplication follows the commutative and associative properties. I can then use the first law of exponents to simplify the expression.

3. An estimate of the number of neurons in the brain of an average rat is 2×10^8. A cat has approximately 8×10^8 neurons. Which animal has a greater number of neurons? By how much?

$$\mathbf{8 \times 10^8 > 2 \times 10^8}$$

I need to divide to figure out how many times larger the cat's number of neurons are than the rat's.

$$\mathbf{\frac{8 \times 10^8}{2 \times 10^8} = \frac{8}{2} \times \frac{10^8}{10^8}}$$
$$\mathbf{= 4 \times 10^{8-8}}$$
$$\mathbf{= 4 \times 10^0}$$
$$\mathbf{= 4 \times 1}$$
$$\mathbf{= 4}$$

The cat has 4 times as many neurons as the rat.

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G8-M1-Lesson 9: Scientific Notation

Definitions

A positive decimal is said to be written in scientific notation if it is expressed as a product $d \times 10^n$, where d is a decimal greater than or equal to 1 and less than 10 and n is an integer.

The integer n is called the order of magnitude of the decimal $d \times 10^n$.

Examples

1. Write the number 32,000,000,000 in scientific notation.

   $\mathbf{32,000,000,000 = 3.2 \times 10^{10}}$

   I will place the decimal between the 3 and 2 to achieve a value that is greater than 1 and smaller than 10.

   I will need to multiply 3.2 by 10^{10} because I need to write an equivalent form of 32,000,000,000.

2. What is the sum of 5.4×10^7 and 8.24×10^9?

   To add terms, they need to be like terms. I know that means that the magnitudes, or the powers, need to be equal.

   $\mathbf{(5.4 \times 10^7) + (8.24 \times 10^9)}$

   $\mathbf{= (5.4 \times 10^7) + \left(8.24 \times (10^2 \times 10^7)\right)}$ ***By first law of exponents***

   $\mathbf{= (5.4 \times 10^7) + \left((8.24 \times 10^2) \times 10^7\right)}$ ***By associative property of multiplication***

   I know that "$\times 10^2$" multiplies 8.24 by 100.

   $\mathbf{= (5.4 \times 10^7) + (824 \times 10^7)}$

   $\mathbf{= (5.4 + 824) \times 10^7}$ ***By distributive property***

   $\mathbf{= 829.4 \times 10^7}$

   $\mathbf{= (8.294 \times 10^2) \times 10^7}$

   $\mathbf{= 8.294 \times 10^9}$

   The last step is to write this in scientific notation.

3. The Lextor Company recently posted its quarterly earnings for 2014.

   Quarter 1: 2.65×10^6 dollars

   Quarter 2: 1.6×10^8 dollars

   Quarter 3: 6.1×10^6 dollars

   Quarter 4: 2.25×10^8 dollars

   What is the average earnings for all four quarters? Write your answer in scientific notation.

$$
\begin{aligned}
\textbf{Average Distance} &= \frac{(2.65 \times 10^6) + (1.6 \times 10^8) + (6.1 \times 10^6) + (2.25 \times 10^8)}{4} \\
&= \frac{(2.65 \times 10^6) + (1.6 \times 10^2 \times 10^6) + (6.1 \times 10^6) + (2.25 \times 10^2 \times 10^6)}{4} \\
&= \frac{(2.65 \times 10^6) + (160 \times 10^6) + (6.1 \times 10^6) + (225 \times 10^6)}{4} \\
&= \frac{(2.65 + 160 + 6.1 + 225) \times 10^6}{4} \\
&= \frac{393.75 \times 10^6}{4} \\
&= \frac{393.75}{4} \times 10^6 \\
&= 98.4375 \times 10^6 \\
&= 9.84375 \times 10^7
\end{aligned}
$$

The average earnings in 2014 for the Lextor Company is 9.84375×10^7 dollars.

G8-M1-Lesson 10: Operations with Numbers in Scientific Notation

1. A lightning bolt produces 1.1×10^{10} watts of energy in about 1 second. How much energy would that bolt of lightning produce if it lasted for 24 hours? (Note: 24 hours is 86,400 seconds.)

$$\mathbf{(1.1 \times 10^{10}) \times 86,400}$$
$$\mathbf{= (1.1 \times 10^{10}) \times (8.64 \times 10^{4})}$$
$$\mathbf{= (1.1 \times 8.64) \times (10^{10} \times 10^{4})}$$
$$\mathbf{= 9.504 \times 10^{10+4}}$$
$$\mathbf{= 9.504 \times 10^{14}}$$

I need to take the amount of energy produced in one second and multiply it by 86,400.

A lightning bolt would produce 9.504×10^{14} watts of energy if it lasted 24 hours.

2. There are about 7,000,000,000 people in the world. In Australia, there is a population of about 2.306×10^{7} people. What is the difference between the world's and Australia's populations?

$$\mathbf{7,000,000,000 - 2.306 \times 10^{7}}$$
$$\mathbf{= (7 \times 10^{9}) - (2.306 \times 10^{7})}$$
$$\mathbf{= (7 \times 10^{2} \times 10^{7}) - (2.306 \times 10^{7})}$$
$$\mathbf{= (700 \times 10^{7}) - (2.306 \times 10^{7})}$$
$$\mathbf{= (700 - 2.306) \times 10^{7}}$$
$$\mathbf{= 697.694 \times 10^{7}}$$
$$\mathbf{= 6.97694 \times 10^{9}}$$

Just like in the last lesson, I need to make sure the numbers have the same order of magnitude (exponent) before I actually subtract.

The difference between the world's and Australia's populations is about 6.97694×10^{9}.

3. The average human adult body has about 5×10^{13} cells. A newborn baby's body contains approximately 2.5×10^{12} cells.

   a. Find their combined cellular total.

   Combined Cells $= \left(5 \times 10^{13}\right) + \left(2.5 \times 10^{12}\right)$

   $$= \left(5 \times 10^{1} \times 10^{12}\right) + \left(2.5 \times 10^{12}\right)$$

   $$= \left(50 \times 10^{12}\right) + \left(2.5 \times 10^{12}\right)$$

   $$= (50 + 2.5) \times 10^{12}$$

   $$= 52.5 \times 10^{12}$$

   $$= 5.25 \times 10^{13}$$

   The combined cellular total is 5.25×10^{13} cells.

   b. Given that the number of cells in the average elephant is approximately 1.5×10^{27}, how many human adult and baby combined cells would it take to equal the number of cells of an elephant?

   $$\frac{1.5 \times 10^{27}}{5.25 \times 10^{13}} = \frac{1.5}{5.25} \times \frac{10^{27}}{10^{13}}$$

   $$\approx 0.286 \times 10^{27-13}$$

   $$= 0.286 \times 10^{14}$$

   $$= 2.86 \times 10^{13}$$

   It would take 2.86×10^{13} adult and baby combined cells to equal the number of cells of an elephant.

G8-M1-Lesson 11: Efficacy of Scientific Notation

1. Which of the two numbers below is greater? Explain how you know.

   8.25×10^{15} and 8.2×10^{20}

   The number $\mathbf{8.2 \times 10^{20}}$ is greater. When comparing each numbers order of magnitude, it is obvious that $\mathbf{20 > 15}$; therefore, $\mathbf{8.2 \times 10^{20} > 8.25 \times 10^{15}}$.

   To figure out which number is greater, I need to look at the order of magnitude (exponent) of each number.

2. About how many times greater is 8.2×10^{20} compared to 8.25×10^{15}?

$$\begin{aligned}\frac{8.2 \times 10^{20}}{8.25 \times 10^{15}} &= \frac{8.2}{8.25} \times \frac{10^{20}}{10^{15}} \\ &= 0.993939\ldots \times 10^{20-15} \\ &\approx 0.99 \times 10^{5} \\ &= 99{,}000\end{aligned}$$

   $\mathbf{8.2 \times 10^{20}}$ is about $\mathbf{99{,}000}$ times greater than $\mathbf{8.25 \times 10^{15}}$.

3. Suppose the geographic area of Los Angeles County is $4{,}751$ sq. mi. If the state of California has area 1.637×10^{5} square miles, that means that it would take approximately 35 Los Angeles Counties to make up the state of California. As of 2013, the population of Los Angeles County was 1×10^{7}. If the population were proportional to area, what would be the population of the state of California? Write your answer in scientific notation.

$$\begin{aligned}1 \times 10^{7} \times 35 &= 35 \times 10^{7} \\ &= (3.5 \times 10) \times 10^{7} \\ &= 3.5 \times (10 \times 10^{7}) \\ &= 3.5 \times 10^{8}\end{aligned}$$

   The population of California is $\mathbf{3.5 \times 10^{8}}$.

   Since it takes about 35 Los Angeles Counties to make up the state of California, then what I need to do is multiply the population of Los Angeles County by 35.

   The expression 35×10^{7} is not in scientific notation because 35 is too large (it has to be less than 10). I can rewrite 35 as 3.5×10 because $35 = 3.5 \times 10$.

G8-M1-Lesson 12: Choice of Unit

1. What is the average of the following two numbers?
   3.257×10^3 and 3.1×10^3

   The average is

$$\frac{3.257 \times 10^3 + 3.1 \times 10^3}{2} = \frac{(3.257 + 3.1) \times 10^3}{2} = \frac{6.357 \times 10^3}{2} = \frac{6.357}{2} \times 10^3 = 3.1785 \times 10^3$$

To find the average, I need to add the two numbers and then divide by 2. Since the numbers are raised to the same power of 10, I really only need to add 3.257 and 3.1.

2. Assume you are given the data below and asked to decide on a new unit in order to make comparisons and discussions of the data easier.

1.9×10^{15}	3.75×10^{19}
9.26×10^{16}	7.02×10^{19}
4.56×10^{17}	2.4×10^{3}

I need to examine the exponents to see which is most common or which exponent most numbers would be close to. Since I'm deciding the unit, I just need to make sure my choice is reasonable.

a. What new unit would you select? Name it and express it using a power of 10.

I would choose to use 10^{18} as my unit. I'm ignoring the number with 10^3 because it is so much smaller than the other numbers. Most of the other numbers are close to 10^{18}. I will name my unit Q.

b. Rewrite at least two pieces of data using the new unit.

$$\frac{1.9\times10^{15}}{10^{18}} = 1.9 \times 10^{15-18} = 1.9 \times 10^{-3} = 0.0019$$

1.9×10^{15} ***rewritten in the new unit is*** $0.0019Q$.

$$\frac{7.02\times10^{19}}{10^{18}} = 7.02 \times 10^{19-18} = 7.02 \times 10^{1} = 70.2$$

7.02×10^{19} ***rewritten in the new unit is*** $70.2Q$.

To rewrite the data, I will take the original number and divide it by the value of my unit, Q, which is 10^{18}.

G8-M1-Lesson 13: Comparison of Numbers Written in Scientific Notation and Interpreting Scientific Notation Using Technology

1. If $a \times 10^n < b \times 10^n$, what are some possible values for a and b? Explain how you know.

   When two numbers are each raised to the same power of $\mathbf{10}$, in this case the power of $\boldsymbol{n}$, then you only need to look at the numbers $\boldsymbol{a}$ and $\boldsymbol{b}$ when comparing the values (Inequality (A) guarantees this). Since we know that $\boldsymbol{a \times 10^n < b \times 10^n}$, then we also know that $\boldsymbol{a < b}$. Then a possible value for $\boldsymbol{a}$ is $\mathbf{5}$ and a possible value for $\boldsymbol{b}$ is $\mathbf{6}$ because $\mathbf{5 < 6}$.

2. Assume that $A \times 10^{-5}$ is not written in scientific notation and A is positive. That means that A is greater than zero but not necessarily less than 10. Is it possible to find a number A so that $A \times 10^{-5} < 1.1 \times 10^5$ is not true? If so, what number could A be?

   Since $\mathbf{10^{-5} = 0.00001}$ and $\mathbf{1.1 \times 10^5 = 110000}$, then a number for $\boldsymbol{A}$ bigger than $\mathbf{1.1 \times 10^{10}}$ would show that $\boldsymbol{A \times 10^{-5} < 1.1 \times 10^5}$ is not true.

   If $\boldsymbol{A = 1.1 \times 10^{10}}$, then by substitution

$$\begin{aligned} \boldsymbol{A \times 10^{-5}} &= \mathbf{(1.1 \times 10^{10}) \times 10^{-5}} \\ &= \mathbf{1.1 \times 10^{10+(-5)}} \\ &= \mathbf{1.1 \times 10^5}. \end{aligned}$$

   If $\boldsymbol{A = 1.1 \times 10^{10}}$, then $\boldsymbol{A \times 10^{-5} = 1.1 \times 10^5}$. Therefore $\boldsymbol{A}$ can be any number as long as $\boldsymbol{A > 1.1 \times 10^{10}}$.

   If $A \times 10^{-5} < 1.1 \times 10^5$ is not written in scientific notation, it means that A can be a really large number. What I'm really being asked is if there is a number I can think of that when multiplied by 10^{-5} or its equivalent $\frac{1}{100000}$ would be larger than 1.1×10^5?

3. Which of the following two numbers is greater?

   2.68941×10^{27} or 2.68295×10^{27}

   Since $\mathbf{2.68941 > 2.68295}$, then $\mathbf{2.68941 \times 10^{27} > 2.68295 \times 10^{27}}$.

   Since both numbers are raised to the same power (Inequality (A) again), all I need to compare is 2.68941 and 2.68295.